Local Subsystems on the Light Front: Luminosity and Local Amplitudes

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joint work with Charalampos Theofilis

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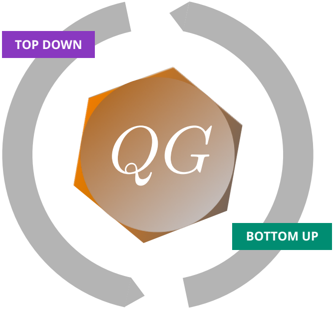
Outline: unscrewing QG



- Top-down: Framework for foundations of local amplitudes.
- Bottom-up: Non-perturbative quantisation of gravitational subsystem at local null hypersurfaces.
- Results: How loop quantum discreteness of geometry can create a bound on radiated power.

Results here build and connect to much earlier and recent work ...

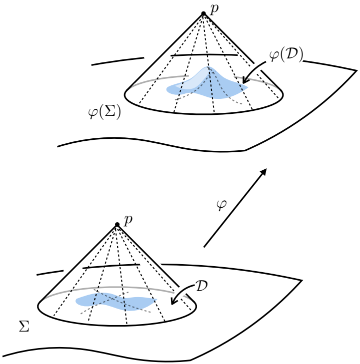
Null-boundary version of covariant LQG [Engle, Rovelli, Pereira, Livine, Freidel, ..., Han, ..., Vidotto, ... Dittrich, Steinhaus, Haggard, Borissova, Asante, ...], quantum null surfaces [Eccles, Kirklin, Freidel, Giambelli], isolated horizons, characteristic null initial data [Ashtekar, Speziale, Lewandowski, Reisenberger,...], quantum reference frames [Brukner, Giacomini, Castro-Ruiz, Chen, ..., Hoehn, Carrozza, ... Kabel, Giesel] .

INTRODUCTION:
Local Amplitudes from Local Subsystems

joint work with Charalampos Theofilis

Take compact region $\mathscr{D}$ on some spacelike initial surface Σ.
What knows an observer at p about the state at Σ?

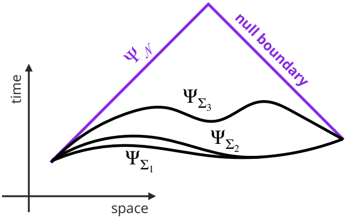
- State of the system = point in the space of solutions
- space of solutions = initial data on Cauchy surface
- equivalence principle implies (passive) coordinate invariance $x^\mu \to \tilde{x}^\mu = x^\mu \circ \varphi$ implies (active) diffeo invariance $\forall \varphi \in \mathrm{Diff}(\mathscr{M} : \mathscr{M})$.
- For φ sufficiently small, what happens at p can be determined by data on $\mathscr{D}$ or $\varphi(\mathscr{D})$.



- State on $\mathscr{D}$ and $\varphi(\mathscr{D})$ are (gauge) equivalent.
- Hence there must be (at least) four constraints $\mathscr{H}_\mu \approx 0$ on the kinematical phase space on $\mathscr{D}$.

Four constraints, one of them is very hard. How to get rid of the Wheeler–De Witt (WDW) equation

- Initial data: three-metric h_{ab} and extrinsic curvature $\tilde{\pi}^{ab} \sim K_{ab} \sim \dot{h}_{ab}$.
- Constraints $\mathscr{H}[h, \tilde{\pi}] = 0$ and $\mathscr{H}_a[h, \tilde{\pi}] = 0$ generate gauge redundancies on phase space.
- Gauge redundancies: states on Σ_1, Σ_2, ... are gauge equivalent.

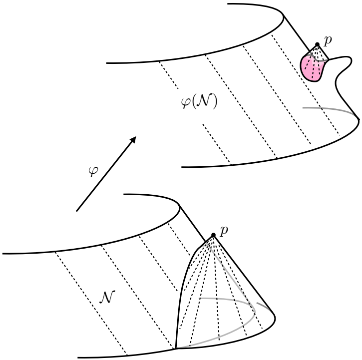


- Basic idea: Characterize the entire gauge equivalence class $[\Psi_{\Sigma_i}]$ by pushing the time-evolution (gauge) to its extreme.
- Boundary of the future Cauchy development of Σ_i is light-like.
- Merely gauge fixing, as in e.g. *shape dynamics*.

Less constraints for null initial data

Take compact region $\mathscr{N}$ on null surface as your subsystem.
Intuitive reason why there are less constraints.

- What happens at p is not fully determined by data on $\mathscr{N}$.
- Take $\varphi \in \mathrm{Diff}(\mathscr{M} : \mathscr{M})$ (compact support) that maps $\mathscr{N}$ into a new surface that is no longer null (but almost everywhere).
- Data on $\varphi(\mathscr{N})$ can now determine what happens at p.
- States on $\varphi(\mathscr{N})$ and $\mathscr{N}$ can no longer be gauge equivalent.



- Otherwise we would have a constraint C_ξ whose flow generates new data on $\varphi(\mathscr{N})$ from data on $\mathscr{N}$. But that data is arbitrary ↯

Take a diffeomorphism invariant field theory with Lagrangian L:

$$\forall f \in \mathrm{Diff}(\mathscr{M} : \mathscr{M}) : f^* L[\varphi^{\mathfrak{a}}, \pi^{\mathfrak{a}}] = L[f^* \varphi^{\mathfrak{a}}, f^* \pi^{\mathfrak{a}}].$$

for p and $(p+1)$-form fields $\varphi^{\mathfrak{a}}$ (configuration variable) and $\pi^{\mathfrak{a}} = \mathrm{d}\varphi^{\mathfrak{a}}$ (kinetic momentum).

Euler–Lagrange equations ($\forall \delta : E(\delta) = 0$) and pre-symplectic potential

$$\delta[L]\Big|_{\pi^{\mathfrak{a}} = \mathrm{d}\phi^{\mathfrak{a}}} = E(\delta) + \mathrm{d}(\vartheta(\delta)),$$

where the boundary term determines the pre-symplectic potential

$$\vartheta(\delta) = \delta\phi^{\mathfrak{b}}(x) \wedge \frac{\partial L[\varphi^{\mathfrak{a}}, \pi^{\mathfrak{a}}](x)}{\partial \pi^{\mathfrak{b}}}\Bigg|_{\pi^{\mathfrak{a}} = \mathrm{d}\varphi^{\mathfrak{a}}}.$$

Since the Lagrangian is invariant under diffeomorphisms, we obtain a conserved current (Noether theorem)

$$j_\xi = \vartheta(\mathscr{L}_\xi) - \xi \lrcorner L.$$

The Noether current is conserved

$$\mathrm{d}j_\xi = 0.$$

By Poincaré's Lemma: $j_\xi = \mathrm{d}q_\xi$. Fluxes and charges

$$H_\xi[\mathscr{N}] := \int_{\mathscr{N}} j_\xi,$$
$$Q_\xi[\mathscr{C}] := \int_{\mathscr{C}} q_\xi.$$

Constraints as flux balance laws

$$C_\xi[\mathscr{N}] = H_\xi[\mathscr{N}] - Q_\xi[\mathscr{C}_+] + Q_\xi[\mathscr{C}_-] \overset{!}{=} 0.$$

Solving quantum gravity amounts to finding physical states

$$\boxed{\forall \xi : \hat{C}_\xi[\mathscr{N}]\Psi_{phys} = 0.}$$

Corner charges and kinematical Hilbert space

Constraints defined on auxiliary (kinematical) Hilbert space

$$K_{\mathcal{N}} = \mathscr{K}_{\mathscr{C}_+} \otimes \mathscr{K}_{\mathcal{N}} \otimes \mathscr{K}_{\mathscr{C}_-}.$$

Kinematical boundary states:

$$|\Psi\rangle \equiv \sum\!\!\!\!\!\!\int_{AA'} |A\rangle \otimes |\Psi^A_{A'}\rangle \otimes \langle A'| \in K_{\mathcal{N}},$$

Edge Hilbert spaces carry representation of Noether charges

$$Q^A{}_{B\xi}[\mathscr{C}] = \langle A|Q_\xi[\mathscr{C}]B\rangle.$$

Projector onto physical states (index position encodes here differnet orientations of $\mathcal{N}$)

$$\begin{aligned} \boldsymbol{P} : &|\gamma^A{}_{A'}\rangle \to \boldsymbol{P}|\gamma^A{}_{A'}\rangle := P^{AB'}{}_{A'B}\,|\gamma^B{}_{B'}\rangle, \\ &|\gamma_{A'}{}^A\rangle \to \boldsymbol{P}|\gamma^A{}_{A'}\rangle := P^{AB'}{}_{A'B}\,|\gamma_{B'}{}^B\rangle. \end{aligned}$$

Projector is an intertwiner for the flux balance laws

The Ward identities $\hat{C}_\xi \boldsymbol{P} = 0 = \boldsymbol{P}\hat{C}_\xi$ are

$$Q^{A}{}_{C\xi}[\mathscr{C}_+]P^{CB'}{}_{A'B} - Q^{C'}{}_{A'\xi}[\mathscr{C}_-]P^{AB'}{}_{C'B} = H_\xi[\mathscr{N}]P^{AB'}{}_{A'B}\,,$$
$$P^{AB'}{}_{A'C}\,Q^{C}{}_{B\xi}[\mathscr{C}_+] - P^{AC'}{}_{A'A}\,Q^{B'}{}_{C'\xi}[\mathscr{C}_-] = P^{AB'}{}_{A'B}\,H_\xi[\mathscr{N}],$$

The charges are observables themselves. Hence they commute with the projector. This means

$$Q^{A}{}_{C\xi_+}[\mathscr{C}_+]P^{CB'}{}_{A'B} - P^{AB'}{}_{A'C}\,Q^{C}{}_{B\xi_+}[\mathscr{C}_+] = 0,$$
$$P^{AC'}{}_{A'B}\,Q^{B'}{}_{C'\xi_-}[\mathscr{C}_-] - Q^{C'}{}_{A'\xi_-}[\mathscr{C}_-]P^{AB'}{}_{C'B} = 0.$$

All this is just to say that $P^{AB'}{}_{A'B}$ is an intertwiner for the boundary symmetry group.

Promise of the Wheeler–De Witt equation: All physics to be extracted from this projector, observables, (asymptotic) S-matrix elements, etc. *How could this work out in practice?*

Bulk and boundary vacuum

Corner (boundary) vacuum

$$\forall \xi : Q^{A}{}_{B\xi}[\mathscr{C}]T^{B} = 0.$$

Bulk vacuum (so it exists) has properties

- Is physical state:

$$\forall \xi : H_{\xi}[\mathscr{N}]|\Omega^{A}{}_{A'}\rangle = Q^{A}{}_{B\xi}[\mathscr{C}_{+}]|\Omega^{B}{}_{A'}\rangle - Q^{B'}{}_{A'\xi}[\mathscr{C}_{-}]|\Omega^{A}{}_{B'}\rangle.$$

- Has vanishing flux:

$$\forall \xi_{+} \exists \xi_{-} : Q^{A}{}_{B\xi_{+}}[\mathscr{C}_{+}]|\Omega^{B}{}_{A'}\rangle = Q^{B'}{}_{A'\xi_{-}}[\mathscr{C}_{-}]|\Omega^{A}{}_{B'}\rangle$$

Local Amplitudes from Quantum Characteristic Gluing

Promise of the Wheeler–De Witt equation: no time evolution, all dynamics to be extracted from physical states.

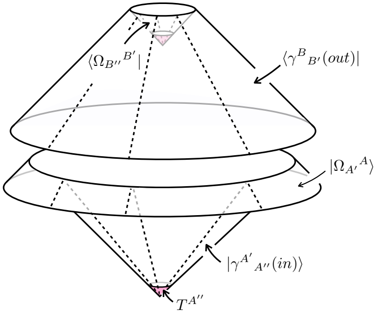
- One incarnation: Covariant LQG, spinfoams

$$W[\Psi_{\partial\mathscr{M}}] = \langle\psi_{out}|\boldsymbol{P}|\psi_{in}\rangle.$$

- Kinematical states on a null slab

$$|\gamma^{A}{}_{A'}\rangle \in \mathscr{K}_{edge} \otimes \mathscr{K}_{bulk} \otimes \mathscr{K}'_{edge}.$$

- Assuming existence of *tip states* T^A, vacuum states $|\Omega^A{}_{A'}\rangle$ and the projector $\boldsymbol{P}$, we can formally introduce local amplitudes.



- Basic idea: Add infinitesimally thin null slabs to upper and lower cones. Data on *in* and *out* folded zig-zag cones diffeomorphic. *Suggests proposal:*

$$\boldsymbol{A}(\gamma(in) \to \gamma(out)) = \langle\Omega,\uparrow| \otimes \langle\gamma(out),\downarrow|\boldsymbol{P}|\gamma(in),\uparrow| \otimes |\Omega,\downarrow\rangle,$$

$$gravity\ amplitudes = \begin{array}{c} contractions\ of\ intertwiner\ \boldsymbol{P}\ with \\ invariant\ bulk\ \Omega\ and\ corner\ states\ T^A. \end{array}$$

Strategy: Quasi-local amplitudes from characteristic gluing [Aretakis, Czimek, Rodnianski; Chruściel].

Basic Properties of the Amplitude Map

It is then easy to show that this amplitude satisfies basic properties

- Projector onto physical states:

$$\boldsymbol{A}\big(H_\xi[\mathscr{N}_{in}]\gamma(in) \to \gamma(out)\big) = \boldsymbol{A}\big(Q_\xi[\mathscr{C}]\gamma(in) \to \gamma(out)\big).$$

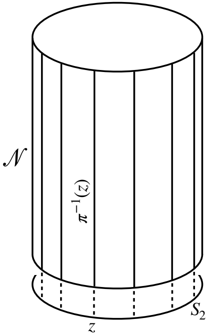
- Charge matching condition:

$$\forall\xi : \boldsymbol{A}\big(Q_\xi[\mathscr{C}]\gamma(in) \to \gamma(out)\big) = \boldsymbol{A}\big(\gamma(in) \to Q_\xi[\mathscr{C}]\gamma(out)\big).$$

How to make these vague ideas concrete:
Bottom-up approach to local amplitudes

Spacetime region bounded by null surface:

- Compact spacetime region $\mathscr{M}$.
- Bounded by spacelike disks M_0, M_1 and null surface $\mathscr{N}$.
- Null surface boundary $\mathscr{N}$ embedded into abstract bundle (ruled surface) $P(\pi, \mathscr{C}) \simeq \mathbb{R} \times \mathscr{C}$.
- Null generators $\pi^{-1}(z)$.



See also work by [Kirklin, Freidel].

Signature $(0{+}{+})$ metric.

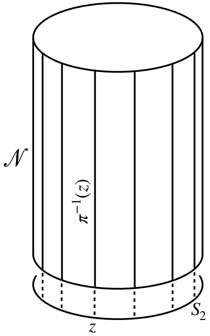
$$\varphi^*_{\mathcal{N}} g_{ab} = q_{ab} = \delta_{ij} e^i{}_a e^j{}_b, \qquad i,j = 1,2.$$

Parametrisation of the dyad

$$\boxed{e^i = \Omega\, S^i{}_m\, e^m_{(o)}.}$$

- Conformal factor Ω parametrizes the overall scale.
- $SL(2,\mathbb{R})$-Holonomy $S^i{}_m$ determines the shape degrees of freedom.
- Fiducial background dyad $e^j_{(o)}$, e.g.

$$\left(e^1_{(o)}, e^2_{(o)}\right) = \left(\mathrm{d}\vartheta, \sin\vartheta\, \mathrm{d}\varphi\right).$$



We consider a null strip $\mathscr{N}$ with two corners as our subsystem. No unique clock along $\mathscr{N}$. Convenient choice

Boundary condition,

$$\mathscr{U}\big|_{\partial\mathscr{N}} = \pm 1.$$

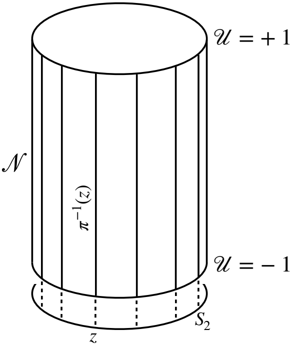
Affinity proportional to expansion

$$\partial^b_{\mathscr{U}}\nabla_b\partial^a_{\mathscr{U}} = -\frac{1}{2}(\Omega^{-2}\frac{\mathrm{d}}{\mathrm{d}\mathscr{U}}\Omega^2)\partial^a_{\mathscr{U}}.$$

Parametrize physical clock $\mathscr{U}$ relative to unphysical coordinate u.

$$\partial_u\mathscr{U} = \mathrm{e}^{\chi}.$$

The *chronoton* χ becomes a quantum reference frame (part of phase space).



[WW arXiv:2104.05803; Giambelli, Feidel, Leigh arXiv:2309.03932, ...]

Diffeomorphisms with compact support on $\mathscr{N}$ are gauge redundancies.

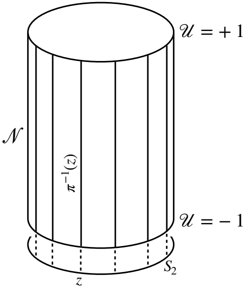
- Remove them by only considering fibre preserving diffeos.
- Residual diffeomorphisms: angle dependent reparametrizations of $\mathscr{U}$.
- Canonical generator on phase space: Raychaudhuri equation.

Raychaudhuri equation

$$\frac{\mathrm{d}^2}{\mathrm{d}\mathscr{U}^2}\Omega^2 = -2\,\sigma\bar{\sigma}\,\Omega^2 \mathrm{e}^{-2\chi}.$$

$SL(2,\mathbb{R})$ holonomy

$$\frac{\mathrm{d}}{\mathrm{d}u} S = \Big(\varphi J + \big(\sigma \bar{X} + \mathrm{cc.}\big)\Big) S.$$



$SL(2,\mathbb{R})$ generators split into $U(1)$ complex structure J and shear generators:

$$[J, X] = -2\mathrm{i}\,X, \qquad [J, \bar{X}] = +2\mathrm{i}\,\bar{X}, \qquad [X, \bar{X}] = \mathrm{i}\,J.$$

Phase space and action

In $D = 4$, there are two Lorentz scalars that we can build from the curvature tensor:

$$R[A, e] = F^{\alpha\beta}{}_{ab}[A] e_\alpha{}^a e_b{}^\beta,$$
$$R^*[A, e] = \frac{1}{2}\varepsilon^{\alpha\beta\mu\nu} F_{\alpha\beta ab}[A] e_\mu{}^a e_\nu{}^b \approx 0.$$

Therefore, in the first-order formalism, there are *two coupling constants* at linear order in the curvature,

$$S = \frac{1}{16\pi G}\int_{\mathscr{M}} d^4v \Big[R - \frac{1}{\gamma} R^*\Big] + \text{boundary terms.}$$

G is Newton's constant, γ is the Barbero–Immirzi parameter.

How does γ affect the quantisation of charges and radiation?

See also [Giesel, Kabel, WW `arXiv:2410.17339`].

Phase space and symplectic structure

Starting from the γ-action for GR, we obtain null symplectic structure

$$\begin{aligned}\Theta_{\mathcal{N}} = \frac{1}{2\mathrm{i}}\int_{\partial\mathcal{N}} d^2v_o\,(a\mathbb{d}\bar{a} - \mathrm{cc.}) + \\ + \int_{\mathcal{N}} d^3v_o\,p_\chi \mathbb{d}\chi + \frac{1}{2\mathrm{i}}\int_{\mathcal{N}} d^3v_o\,\big(b\mathbb{d}\bar{b} - \mathrm{cc.}\big) + \\ + \int_{\mathcal{N}} d^3v_o\,\mathrm{Tr}_{\mathfrak{sl}(2,\mathbb{R})}\big(\Pi\mathbb{d}SS^{-1}\big)\,.\end{aligned}$$

Geometric interpretation:

- Area operator becomes number operator: $\Omega^2\big|_{\mathscr{C}_\pm} = 8\pi\gamma G\,a\bar{a}$.
- Expansion is number operator too: $\frac{\mathrm{d}}{\mathrm{d}u}\Omega^2 = 8\pi\gamma G\,b\bar{b}$.
- Chronoton modes: $p_\chi = \frac{1}{8\pi G}\,\frac{\mathrm{d}}{\mathrm{d}u}\Omega^2$ (a constraint, as $K = \gamma L$ in LQG).
- Last line: $\mathfrak{su}(1,1)$ shape modes.
- GR phase space: these variables plus constraints (all polynomial).

[WW arXiv:2104.05803, arXiv:2504.10802, ...]

New canonical variables

Among canonical variables are *shape modes* $S \in SL(2,\mathbb{R})/U(1) \simeq SU(1,1)/U(1)$.

- The conjugate momentum is $\Pi^A{}_B \in \mathfrak{su}(1,1)$.

$$\Pi = -\frac{1}{16\pi\gamma G}\frac{\mathrm{d}}{\mathrm{d}u}\Omega^2 J - \frac{1}{8\pi\gamma G}\big((\gamma+\mathrm{i})\sigma\bar{X} + (\gamma-\mathrm{i})\bar{\sigma}X\big) \in \mathfrak{su}(1,1).$$

- Utilize (fermionic) bosonic representation

$$\Pi_{AB} = \pi_{(A}\omega_{B)} \equiv \frac{1}{2}\left(\pi_A\omega_B + \pi_B\omega_A\right).$$

- Fundamental Poisson brackets

$$\begin{aligned}\{\pi_A(u,\zeta,\bar{\zeta}), \omega_B(u',\zeta,\bar{\zeta})\} &= +\epsilon_{AB}\delta_{\mathcal{N}}(u-u',\zeta-\zeta',\bar{\zeta}-\bar{\zeta}'),\\ \{\underset{\sim}{\pi}_A(u,\zeta,\bar{\zeta}), \underset{\sim}{\omega}_B(u',\zeta,\bar{\zeta})\} &= -\epsilon_{AB}\delta_{\mathcal{N}}(u-u',\zeta-\zeta',\bar{\zeta}-\bar{\zeta}').\end{aligned}$$

- Reconstruction of $S \in SU(1,1)$

$$\underset{\sim}{\omega}^A = [S^{-1}]^A{}_B\omega^B, \qquad \underset{\sim}{\pi}^A = [S^{-1}]^A{}_B\pi^B.$$

Hamilton constraint

There are first-class and second-class constraints. Among the first-class constraints is the Raychaudhuri constraint, which is

$$H[N] = \frac{\mathrm{i}}{2}\int_{\mathcal{N}} d^3v_o\, N(\bar{b}\dot{b} - \mathrm{cc.}) + \int_{\mathcal{N}} d^3v_o\, p_\chi(N\dot{\chi} + \dot{N}) + \\ + \frac{1}{2}\int_{\mathcal{N}} d^3v_o\, N\left(\pi_A\dot{\omega}^A - \dot{\pi}_A\omega^A - \underset{\sim}{\pi}_A\dot{\underset{\sim}{\omega}}^A + \dot{\underset{\sim}{\pi}}_A\underset{\sim}{\omega}^A\right).$$

Basic idea:

1. The Raychaudhuri constraint $H[N]$ generates a Virasoro algebra.
   See also recent results by Freidel and Ciambelli.
2. Idea: Utilize CFT methods to quantize this algebra.
   Mode expansion, positive (negative) frequency modes, Fock vacuum etc.
3. Problem: What selects the statistics of the oscillators?

Ashtekar–Lewandowski vacuum

Introduce tessellation of null surface cuts (thickend null rays).

- Smearing $(p_\chi, \chi, b, \pi_A, \omega^A \dots)$

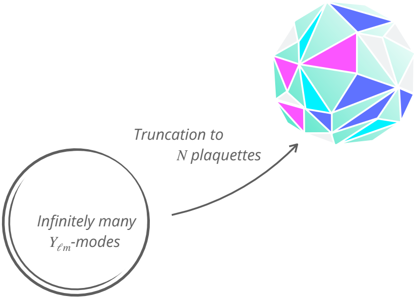
$$p_\chi(i) = \int_{\mathscr{C}_i} d^2 v_o \, p_\chi,$$

$$\chi(i) = \chi(x_i), \quad x_i \in \mathscr{C}_i, \quad \textit{etc.}$$

- Mode expansion

$$\chi(i) = \frac{1}{\sqrt{2\pi}} \sum_{n=-\infty}^{\infty} \chi_n(i) \mathrm{e}^{-inu},$$

$$p_\chi(i) = \frac{1}{\sqrt{2\pi}} \sum_{n=-\infty}^{\infty} p_{\chi n}(i) \mathrm{e}^{-inu}.$$



Ashtekar–Lewandowski (no geometry) vacuum for all modes, e.g.

$$\boxed{\chi_n(i)|0\rangle = p_{\chi n}(i)|0\rangle = \dots = 0, \quad n > 0, i = 1, \dots, N.}$$

Construction builds on earlier ideas by Thomas Thiemann on quantization of half-densities [Thiemann `arXiv:gr-qc/9705021`].

$SU(1,1)$ Casimir

Canonical momentum dual to the shape modes:

$$\Pi = LJ + c\bar{X} + \bar{c}X \in \mathfrak{su}(1,1)$$

$SU(1,1)$ Casimir in terms of the geometric data:

$$L^2 - c\bar{c} = \frac{1}{(16\pi\gamma G)^2}\Omega^4\left(\vartheta^2 - 4(1+\gamma^2)\sigma\bar{\sigma}\right).$$

What we find is:

- Bose statistics for $(\pi_A, \omega^A, b, \bar{b})$:
  - CFT has negative central charge.
  - Both $L^2 \leq c\bar{c}$ and $L^2 \geq c\bar{c}$ possible.
  - But resulting CFT is non-unitary.
- Fermi statistics for $(\pi_A, \omega^A, b, \bar{b})$:
  - CFT has positive central charge.
  - Only $L^2 \geq c\bar{c}$ *infra-Planckian* modes occur.
  - violation of unitarity can be avoided.

On physical grounds (unitarity), we are led to choose Fermi statistics.

It is easy to check that this implies the inequality

$$\vartheta^2 - 4(1+\gamma^2)\sigma\bar{\sigma} \geq 0.$$

For semi-classical states, we should thus get (as expectation values)

$$\frac{\sigma\bar{\sigma}}{\vartheta^2} \leq \frac{1}{4}\frac{1}{1+\gamma^2}.$$

This must hold for all null hypersurfaces.

Caveat: We do not have constructed such semi-classical states explicitly.
In here, we merely assume they exist.

Utilize Bondi expansion to characerize gravitational radiation.

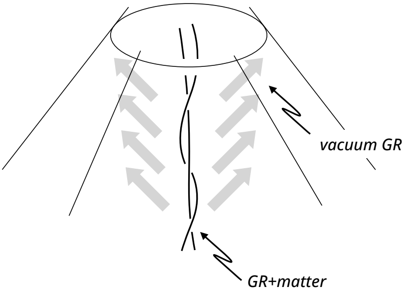
- Bondi mass loss formula

$$\dot{M}_{\mathrm{B}}(u) = -\frac{1}{4\pi G}\oint_{S^2_u\subset\mathscr{I}_+} d^2v_o\,\dot{\sigma}^{(0)}\dot{\bar{\sigma}}^{(0)}.$$

- Falloff conditions

$$\sigma_{(\ell)}(u,r,\zeta,\bar{\zeta}) = -\frac{\dot{\sigma}^{(0)}(u,\zeta,\bar{\zeta})}{r} + \mathcal{O}(r^{-2}),$$

$$\vartheta_{(\ell)}(u,r,\zeta,\bar{\zeta}) = -\frac{2}{r} + \mathcal{O}(r^{-2}).$$



Asymptotic luminosity from quasi-local observables

$$\mathscr{L}_{\mathrm{B}}(u,\zeta,\bar{\zeta}) = \frac{4c^5}{G}\lim_{r\to\infty}\frac{\bar{\sigma}_{(\ell)}(u,r,\zeta,\bar{\zeta})\sigma_{(\ell)}(u,r,\zeta,\bar{\zeta})}{(\vartheta_{(\ell)}(u,r,\zeta,\bar{\zeta}))^2} \leq \frac{c^5}{G}\frac{1}{1+\gamma^2}.$$

In the S-matrix approach, the $\mathcal{O}(r^{-1})$ term in $\vartheta_{(\ell)}$ is a commuting c-number. In the quasi-local quantisation of gravity, it becomes a q-number akin to LQG area operator.

Planck scale luminosity

Humanity has come close to observing such power

$$\mathscr{L}_{\mathrm{P}} = \frac{c^5}{G} \approx 3,63 \times 10^{52}\,\mathrm{W},$$
$$\mathscr{L}_{peak}\Big|_{\mathrm{GW150914}} \approx 3,6 \times 10^{49}\,\mathrm{W}.$$

Side remark: Only in $D = 4$ spacetime dimensions, the Planck power (luminosity) is independent of $\hbar$

$$\mathscr{L}_{\mathrm{P}} = \frac{m_{\mathrm{P}} c^2}{t_{\mathrm{P}}} = \frac{\hbar^{\frac{D-4}{D-2}} c^{\frac{2D+2}{D-2}}}{G^{\frac{2}{D-2}}}.$$

Conclusion

Take home messages

- Local amplitudes from gluing local subsystems on the light front.
- Non-perturbative quantisation of null initial data at finite distance.
  - Spectra for geometric observables reproduce LQG discretenss of area.
  - Difference of the area at initial and final cut: number operator.
  - Turning on γ, we activate otherwise irrelevant $SU(1,1)$ representations.
- We strengthened earlier conjecture on Planck luminosity bound.
- Results implicitly proof that there are theories of QG in which $r \to \infty$ and $\hbar \leftarrow 0$ may not commute.