Boundary modes and quantum reference frames in linearized gravity

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Workshop, Erlangen

02-05-2023

Outline

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There is a new nascent paradigm in quantum gravity, the paradigm of *quasi-local holography*.

- It is *quasi-local* rather than *local*, because observables are attached to finite regions rather than points on the manifold.
- It is *holographic*, because evolution is studied through the exchange of charges at the boundary seperating system and environment.

Nascent paradigm and nascent community connecting research on quantum reference frames, quantum gravity, observables, holography.

Outline

1. Motivation: open systems and quasi-local holography
2. Metriplectic geometry for gravitational subsystems
3. Decoupling limit for edge modes and boundary charges

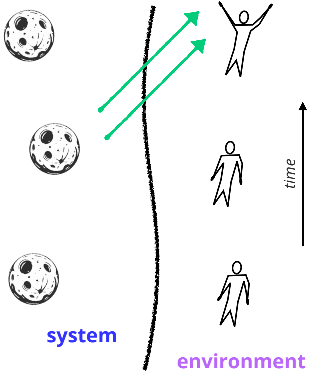
*V. Kabel, Č. Brukner, and ww, Quantum Reference Frames at the Boundary of Spacetime, (2023), arXiv:2302.11629.
*V. Kabel and ww, Metriplectic geometry for gravitational subsystems, Phys. Rev. D (2022), arXiv:2206.00029.
*S. Carrozza, P. A. Hoehn, Edge modes as reference frames and boundary actions from post-selection, JHEP (2022), arXiv:2109.06184.
*A. Vanrietvelde, P. A. Höhn, F. Giacomini, and E. Castro-Ruiz, A change of perspective: switching quantum reference frames via a perspective-neutral framework, Quantum (2020), arXiv:1809.00556 .
*F. Giacomini, E. Castro-Ruiz, Č. Brukner, Quantum mechanics and the covariance of physical laws in quantum reference frames, Nat. Commun. (2019), arXiv:1712.07207.
*William Donnelly, Laurent Freidel, Local subsystems in gauge theory and gravity, JHEP (2016), arXiv:1601.04744.

1. Motivation: open systems and quasi-local holography

In gravity, all systems are open

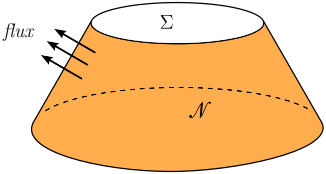
All physics relies on truncations. We separate systems from the environment including observer external. System evolution characterized by relatively small number of coarse-grained observables.

- No gravity shields. No gravity mirrors. No obvious way to isolate subsystems on entire *phase space*.
- No obvious coarse-graining on the gravitational phase space. Averaging problem in cosmology.
- No global Dirac observables on the entire phase space of the theory.



No integrability of charges if the system is open

- In gravity, time evolution $t \to t + \varepsilon$ can be understood as a large gauge transformation.
- Holography suggests to expect that the Hamiltonian is the generator for such a gauge transformation:



$$H[\Sigma] \equiv P_\xi[\Sigma] \stackrel{?}{=} \oint_{\partial\Sigma} d^2v^a\, \xi^b T_{ab}[?].$$

- We assume that P_ξ generates the symmetry algebra

$$\{P_\xi, P_{\xi'}\} = -P_{[\xi,\xi']} + c[\xi, \xi'].$$

- However, that's at odds with the fact that a system may loose mass via gravitational radiation

$$\left.\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t} Mc^2 &= \frac{\mathrm{d}}{\mathrm{d}t} H = \{H, H\} = 0, \\ &= -\frac{1}{4\pi G} \oint_{S^2_t} d^2\Omega\, |\dot{\sigma}^0|^2 \leq 0. \end{aligned}\right\} \lightning$$

- How to deal with this situation? The system is open, no Hamiltonian.

Three possible viewpoints

1. There is no problem: Open systems interact with their environment. So no surprise that there is no Hamiltonian that would measure the gravitational energy in a finite region. End of story.
2. Treat the system as explicitly time-dependent.
   - Time dependence induced by the choice of (outer) boundary conditions.
   - Hamiltonian field equations modified (contact geometry).

$$\frac{\mathrm{d}}{\mathrm{d}t}F_t = \{H, F_t\} + \frac{\partial}{\partial t}F_t.$$

   - By fixing the outgoing flux, radiative data no longer free (highly non-local constraints).
   - Resulting phase space (on which this Hamiltonian operates) is the phase space of edge modes alone. *Seems too restrictive, less useful.*
3. Metriplectic geometry
   - New algebraic approach. New bracket. But many properties of Poisson manifolds lost.
   - Noether charges generate evolution for generic vector fields.
   - Takes into account dissipation.

Bulk plus boundary action

General action for coupled bulk plus boundary field theory

$$S = \int_{\mathscr{M}} L[\Phi, \mathrm{d}\Phi] + \int_{\mathscr{B}} \ell[\Phi, \varphi, \mathrm{d}\varphi|\sigma].$$

Fundamental configuration variables:

- bulk variables: $\Phi \in \Omega^{|\Phi|}(\mathscr{M} : \mathbb{V}_{bulk})$,
- bulk variables: $\varphi \in \Omega^{|\varphi|}(\mathscr{M} : \mathbb{V}_{bndry})$,
- boundary sources: σ, $\delta[\sigma] = 0$,
- Covariance: for every diffeomorphism $\alpha \in \mathrm{Diff}(\mathscr{M} : \mathscr{M})$.

$$L\big[\alpha^*\Phi, \alpha^*\mathrm{d}\Phi\big](x) = L\big[\Phi, \mathrm{d}\Phi\big]\big(\alpha(x)\big),$$
$$\ell\big[\alpha^*\Phi, \alpha^*\varphi, \alpha^*\mathrm{d}\varphi\big|\alpha^*\sigma\big](x) = \ell\big[\Phi, \varphi, \mathrm{d}\varphi\big|\sigma\big]\big(\alpha(x)\big).$$

In $2+1$, the boundary source is simply the conformal metric $\sigma \equiv \sqrt{q}q^{ab}$. In higher dimensions, σ describes also the flux of gravitational radiation crossing the boundary.

Bulk plus boundary field equations

Bulk kinetic momentum: $\Pi_\Phi := \mathrm{d}\Phi$,

Boundary kinetic momentum: $\pi_\phi := \mathrm{d}\phi$,

Pre-symplectic currents:

$$\Theta_{bulk}(\delta) = (-1)^{d-|\Phi|} \frac{\partial L}{\partial \Pi_\Phi} \wedge \delta\Phi \equiv P_\Phi \wedge \delta\Phi,$$
$$\Theta_{edge}(\delta) = (-1)^{d-|\varphi|} \frac{\partial \ell}{\partial \pi_\varphi} \wedge \delta\varphi \equiv p_\varphi \wedge \delta\varphi,$$

Pre-symplectic structure on a partial Cauchy surface Σ,

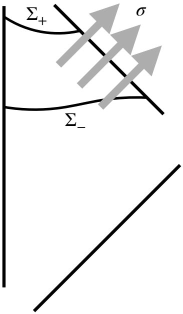
$$\Theta_\Sigma = \int_\Sigma P_\Phi \wedge \mathbb{d}\Phi + \oint_{\partial\Sigma} p_\varphi \wedge \mathbb{d}\varphi.$$

Variation of the action

$$\delta[S] = \Theta_{\Sigma_+}(\delta) - \Theta_{\Sigma_-}(\delta) + \int_{\mathscr{B}} \Theta_{source}(\delta) + \mathrm{EOM}.$$

Pre-symplectic structure on a partial Cauchy surface Σ,

$$\Theta_\Sigma = \int_\Sigma P_\Phi \wedge \mathbb{d}\Phi + \oint_{\partial\Sigma} p_\varphi \wedge \mathbb{d}\varphi,$$
$$\Omega_\Sigma = \mathbb{d}\Theta_\Sigma.$$



Quasi-Hamiltonian via Legendre transformation

$$H_\xi[\Sigma] = \Theta_\Sigma(\mathscr{L}_\xi) - \int_\Sigma \xi \lrcorner L + \oint_{\partial\Sigma} \xi \lrcorner \ell = \oint_{\partial\Sigma} q_\xi.$$

Hamiltonian depends on boundary sources

$$\delta\left[H_\xi[\Sigma]\right] = -\Omega_\Sigma(\mathscr{L}_\xi, \delta) + H_{\delta\xi}[\Sigma] + \oint_{\partial\Sigma} \xi \lrcorner \Theta_{source}(\delta).$$

Too restrictive—constraints (all) radiative data on Σ.

2. Metriplectic approach

Metriplectic geometry

Even dimensional manifold $\mathscr{P}$, equipped with a pre-symplectic two-form $\Omega(\cdot,\cdot) \in \Omega^2(\mathscr{P})$ and a signature (p,q,r) metric tensor $G(\cdot,\cdot)$.

A vector field $\mathfrak{X}_F$ is a (right) Hamiltonian vector field of some (gauge invariant) functional $F : \mathscr{P} \to \mathbb{R}$ on $(\mathscr{P}, \Omega, G)$ iff

$$\forall \delta \in T\mathscr{P} : \delta[F] = \Omega(\delta, \mathfrak{X}_F) - G(\delta, \mathfrak{X}_F).$$

The Leibniz bracket between two such functionals is given by

$$(F, G) = \mathfrak{X}_F[G].$$

The metric on phase space encodes dissipation

$$\frac{\mathrm{d}}{\mathrm{d}t} H = (H, H) = -G(\mathfrak{X}_H, \mathfrak{X}_H).$$

*Morrison; Kaufman (1982-); Grmela, Göttinger (1997); Guha (2002); Holm, Stanley (2003);...

Metriplectic geometry and extended phase space

Following Freidel, Ciambelli, Leigh, we work on an extended pre-symplectic phase space.

A point on the extended pre-symplectic phase space is labelled by a Einstein metric g_{ab} and choice of coordinate functions x^μ.

Maurer – Cartan form for diffeomorphisms

$$\mathbb{X}^a = \left[\frac{\partial}{\partial x^\mu}\right]^a \mathbb{d}x^\mu.$$

Extended pre-symplectic current

$$\delta[L] \approx \mathrm{d}[\vartheta(\delta)],$$

$$\vartheta_{ext} = \vartheta - \vartheta(\mathscr{L}_{\mathbb{X}}) + \mathbb{X} \lrcorner L = \vartheta - \mathrm{d}q_{\mathbb{X}}.$$

Noether charge and Noether charge aspect

$$Q_{\mathbb{X}} = \oint_{\partial\Sigma} q_{\mathbb{X}} = \int_{\Sigma} \big(\vartheta(\mathscr{L}_{\mathbb{X}}) - \mathbb{X} \lrcorner L\big).$$

*L. Freidel, A canonical bracket for open gravitational system, (2021), arXiv:2111.14747.
*L. Ciambelli, R. Leigh, Pin-Chun Pai, Embeddings and Integrable Charges for Extended Corner Symmetry, Phys. Rev. Lett. **128** (2022), arXiv:2111.13181.

The coordinate functions $x^\mu : U \subset \mathscr{M} \to \mathbb{R}^4$ are now part of phase space.

Variations of coordinate functions will only contribute a corner term to the extended pre-symlpleictc two-form.

Vector fields that are determined by their component functions $\xi^\mu(x)$ become field dependent vector fields.

$$\xi^a = \xi^\mu(x)\partial^a_\mu,$$

$$\delta[\xi^a] = [\mathbb{X}(\delta), \xi]^a.$$

Extended pre-symplectic structure on the covariant phase space [Freidel; Ciambelli, Leigh]

$$\Omega_{ext}(\delta_1, \delta_2) = \Omega(\delta_1, \delta_2) + Q_{[\mathbb{X}(\delta_1), \mathbb{X}(\delta_2)]} + \oint_{\partial\Sigma} \mathbb{X}(\delta_{[1}) \lrcorner \vartheta(\delta_{2]})$$

Super metric on phase space [Viktoria Kabel, ww]

$$G(\delta_1, \delta_2) = -\oint_{\partial\Sigma} \mathbb{X}(\delta_{(1}) \lrcorner \vartheta(\delta_{2)})$$

Leibniz bracket on extended phase space,

$$\begin{aligned} \delta[F] &= \Omega_{ext}(\delta, \mathfrak{X}_F) - G(\delta, \mathfrak{X}_F), \\ (F, G) &= \mathfrak{X}_F[G]. \end{aligned}$$

On the extended phase space, the Lie derivative $\mathscr{L}_\xi$ is a Hamiltonian vector field with respect to the Leibniz structure.

The corresponding generator is the Noether charge,

$$\delta[Q_\xi] = \Omega_{ext}(\delta, \mathscr{L}_\xi) - G(\delta, \mathscr{L}_\xi).$$

Leibniz bracket captures dissipation

$$(Q_\xi, Q_\xi) = -\oint_{\partial\Sigma} \xi \lrcorner \vartheta(\mathscr{L}_\xi).$$

But violates Jacobi identity and skew-symmetry of Poisson bracket

$$(A,(B,C)) + (B,(C,A)) + (C,(A,B)) \neq 0,$$
$$(A,B) \neq (B,A).$$

3. Decoupling limit for edge modes and boundary charges

Utility of decoupling limits

Instead of studying edge modes and boundary charges at full non-perturbative level, we consider a simplified problem.

Bulk plus boundary phase space in a decoupling limit $\ell = \sqrt{8\pi G} \to 0$.

- Decoupling limits are useful to understand constituents of phase space in a situation in which it is hard to understand the coupling between all modes of a theory at full non-perturbative level.
- It is a way to linearize phase space. Work on tangent space $T_P\mathscr{P}$ of phase space in a neighbourhood of a fixed solution P.
- A decoupling limit does not change the size of phase space. It can only change the Hamiltonian and how the modes interact.
- Widely used in e.g. massive gravity.

Apply this technique to bulk and boundary phase space of gravity.

*C. de Rham, Massive Gravity, Living Rev. Rel. (2014), arXiv:1401.4173.

First-order variables, action and phase space

Action for the tetrad and the connection

$$S_{grav}[A,e] = \frac{1}{16\pi G}\int_{\mathscr{M}} *(e_\alpha \wedge e_\beta)\wedge F^{\alpha\beta}[A].$$

Coupled to N point particles (caveat: infinite energy density)

$$S\big[A,e,\{\gamma_i,N_i,p_i\}_{i=1}^{N}\big] = \sum_{i=1}^{N}\int_{\gamma_i}\Big(p^i_\alpha e^\alpha - \frac{N_i}{2}\big(p^i_\alpha p^\alpha_i - m_i^2\big)\Big),$$

Symplectic current at non-perturbative level:

$$\vartheta_{abc}(\delta) = \frac{3}{16\pi G}\,\epsilon_{\alpha\beta\gamma\delta}\,e^\alpha{}_{[a}e^\beta{}_b\delta A^{\gamma\delta}{}_{c]} + \sum_{i=1}^{N}\int_{\gamma_i}\mathrm{d}\tau\,\tilde{\delta}^{(4)}_{\gamma_i(\tau)}\,\partial^d_\tau\underset{\sim}{\varepsilon}_{dabc}\,p^i_f\delta\gamma^f_i\,,$$

where $\delta\gamma^a$ is the variation of the path.

Minkowski metric is a solution to the vacuum Einstein equations. There are infinitely many Minkowski geometries

$$\eta_{ab} = \eta_{\mu\nu} \partial_a X^\mu \partial_b X^\nu .$$

Quantum Minkowski space and quantum reference frames $\{X^\mu\}$ are two sides of the same coin.

Minkowski solution for tetrad and connection

$$\begin{aligned} e^\alpha &= \Lambda^\alpha{}_\mu \mathrm{d}X^\mu, & X^\mu &: \mathscr{M} \to \mathbb{R}^4, \\ A^\alpha{}_\beta &= \Lambda^\alpha{}_\mu \mathrm{d}\Lambda_\beta{}^\mu, & \Lambda^\alpha{}_\mu &: \mathscr{M} \to SO(1,3), \end{aligned}$$

Are they all gauge equivalent?

Perturbation theory

Perturbations around Minkoswki space

$$e^\alpha = \Lambda^\alpha{}_\mu\big(\mathrm{d}X^\mu + f^\mu\big),$$
$$A^\alpha{}_\beta = \Lambda^\alpha{}_\mu \mathrm{d}\Lambda_\beta{}^\mu + \Lambda^\alpha{}_\mu \Delta^\mu{}_\nu \Lambda_\beta{}^\mu.$$

Formal power expansion with respect to the coupling constant

$$f^\mu = \sqrt{8\pi G}\,{}^{(1)}f^\mu + 8\pi G\,{}^{(2)}f^\mu + \dots,$$
$$\Delta^\mu{}_\nu = \sqrt{8\pi G}\,{}^{(1)}\Delta^\mu{}_\nu + 8\pi G\,{}^{(2)}\Delta^\mu{}_\nu \dots$$

First order describes free radiation field. Imposing gauge conditions for ${}^{(1)}f^\mu = {}^{(1)}f^\mu{}_\nu \mathrm{d}X^\nu$ to bring perturbation into the standard form

$${}^{(1)}f_{\mu\nu} = \frac{1}{(2\pi)^{\frac{3}{2}}} \int \frac{d^3k}{2|\vec{k}|}\Big(m_\mu m_\nu\, a_+(\vec{k})\mathrm{e}^{\mathrm{i}k_\mu X^\mu} + \bar{m}_\mu \bar{m}_\nu\, a_-(\vec{k})\mathrm{e}^{\mathrm{i}k_\mu X^\mu} + \mathrm{cc.}\Big)$$

Second-order perturbation describes Coulombic fields sourced by effective stress energy tensor (matter+radiation).

$$*\big[\mathrm{d}^{(2)}\Delta\big]^\mu{}_\nu \wedge \mathrm{d}X^\nu = {}^{(2)}T^\mu + {}^{(2)}t^\mu,$$
$$\mathrm{d}^{(2)}f^\mu + {}^{(2)}\Delta^\mu{}_\nu \wedge \mathrm{d}X^\nu = {}^{(2)}\theta^\mu,$$

Perturbative expansion of the pre-symplectic two-form wrt. the coupling constant $\ell = \sqrt{G}$ in region $\mathscr{D}$:

- Bulk plus boundary sympletic structure

$$\Omega_{\mathscr{D}} = \Omega_{\mathscr{D}}^{matter} + \Omega_{\mathscr{D}}^{rad} + \Omega_{\partial\mathscr{D}} + \mathscr{O}(\ell).$$

- Matter contribution

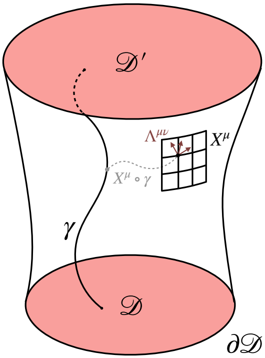
$$\Omega_{\mathscr{D}}^{matter} = \sum_{i=1}^{N} \mathbb{d}p_{\mu}^{i} \left(\mathbb{d}\gamma_i^{\mu} + \mathbb{d}X^{\mu}\big|_{\gamma_i \cap \mathscr{D}}\right).$$

- Radiation modes

$$\Omega_{\mathscr{D}}^{rad} = \int_{\mathscr{D}} * \left(\mathrm{d}X_{[\mu} \wedge \mathbb{D}^{(1)} f_{\nu]}\right) \wedge \mathbb{D}^{(1)} \Delta^{\mu\nu}.$$

- Boundary modes

$$\Omega_{\partial\mathscr{D}} = \oint_{\partial\mathscr{D}} \mathbb{d}P_{\mu}\, \mathbb{d}X^{\mu} - \frac{1}{2} \oint_{\partial\mathscr{D}} \left[\mathbb{d}S^{\mu}{}_{\nu}\, \mathbb{m}^{\nu}{}_{\mu} + \frac{1}{2} S^{\mu}{}_{\nu} \left[\mathbb{m}, \mathbb{m}\right]^{\nu}{}_{\mu}\right]$$



- Maurer Cartan forms in field space:

$$\mathbb{m}^{\mu}{}_{\nu} = \mathbb{d}\Lambda_{\alpha}{}^{\mu}\,\Lambda^{\alpha}{}_{\nu}, \quad \mathbb{X}^{a} = \left[\frac{\partial}{\partial X^{\mu}}\right]^{a} \mathbb{d}X^{\mu}$$

- Field space differential: $\mathbb{D}f^{\mu} = \mathbb{d}f^{\mu} - \mathscr{L}_{\mathbb{X}} f^{\mu}$
- Boundary spin current

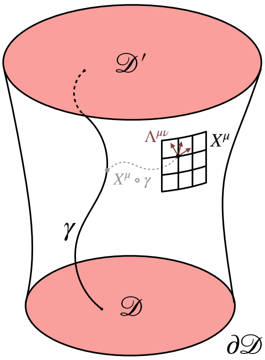
$$S_{\mu\nu} := \frac{1}{8\pi G}\varphi^{*}_{\partial\mathscr{D}}\Big[*\left(\mathrm{d}X_{[\mu}\wedge \mathrm{d}X_{\nu]}\right) + 16\pi G * \left(\mathrm{d}X_{[\mu}\wedge {}^{(2)}f_{\nu]}\right)\Big]$$

- Boundary momentum current

$$P_{\mu} := \varphi^{*}_{\mathscr{C}}\left(\left(*^{(2)}\Delta_{\mu\nu}\right)\wedge \mathrm{d}X^{\nu}\right).$$

- Angular momentum current:

$$J_{\mu\nu} = 2P_{[\mu}X_{\nu]} + S_{\mu\nu}.$$



Creation and annihilation operators for radiation field

$$\left\{a_s(\vec{k}), \bar{a}_{s'}(\vec{k'})\right\} = 2\mathrm{i}|\vec{k}|\delta_{ss'}\delta^{(3)}(\vec{k}-\vec{k}') + \mathcal{O}(\sqrt{8\pi G}), \quad s, s' \in \{\pm\}.$$

Fock vacuum

$$a_s(\vec{k})\big|0\rangle = 0.$$

Kinematical bulk and boundary state space

$$\mathscr{K}_{\mathscr{D}} = \underbrace{\mathscr{H}_{\mathscr{D}}^{matter} \otimes \mathscr{H}_{\mathscr{D}}^{rad}}_{\mathscr{H}_{bulk}} \otimes \mathscr{H}_{\partial\mathscr{D}}^{bndry}.$$

At finite distance with coordinates on the sphere e.g. $\vec{\zeta} = (z, \bar{z})$, we obtain the boundary phase space

$$\big\{P_\mu(\vec{\zeta}), X^\nu(\vec{\zeta}')\big\} = \delta^\nu_\mu \tilde{\delta}_{\mathscr{C}}(\vec{\zeta}, \vec{\zeta}'),$$
$$\big\{S_{\mu\nu}(\vec{\zeta}), \Lambda^\alpha{}_\rho(\vec{\zeta}')\big\} = +2\,\eta_{\rho[\mu}\Lambda^\alpha{}_{\nu]}(\vec{\zeta})\delta_{\mathscr{C}}(\vec{\zeta}, \vec{\zeta}'),$$
$$\big\{S_{\mu\nu}(\vec{\zeta}), S_{\mu'\nu'}(\vec{\zeta}')\big\} = -4\,\delta^\rho_{[\mu}\delta^\sigma_{\nu]}\eta_{\sigma\sigma'}\delta^{\sigma'}_{[\mu'}\delta^{\rho'}_{\nu']}\tilde{S}_{\rho\rho'}(z)\delta_{\mathscr{C}}(\vec{\zeta}, \vec{\zeta}'),$$

Still need to impose constraints that link boundary currents to the fields in the bulk, i.e. a residue of the Wheeler–DeWitt equation. Infinitely many bulk–boundary constraints—conservation laws in any direction

$$C_\mu = P_\mu - \varphi^*_{\mathscr{C}}\Big(\big({*}^{(2)}\Delta_{\mu\nu}\big) \wedge \mathrm{d}X^\nu\Big) = 0.$$

Similar spin constraints $C_{\mu\nu} = -C_{\nu\mu}$ for internal Lorentz rotations.

Relation to asymptotic symmetries

At spacelike infinity $\rho = \sqrt{X_\mu X^\mu} \to \infty$, algebra of currents simplifies, e.g. $\{P_\mu(\vec{\zeta}), P_\nu(\vec{\zeta})\} = 0$.

Conjugate variable, i.e. embedding functions $X^\mu(\zeta)$, diverge.

Regularisation

- introduce fiducial coordinate frame X_o^μ kept fixed $\delta X_o^\mu = 0$
- embedding functions

$$X^\mu(\vec{\zeta}) = \Omega^\mu{}_\nu X_0^\nu + \underbrace{Q^\mu(\vec{\zeta})}_{\textit{angle-dependent translation}}.$$

- $Q^\mu(\vec{\zeta})$ generates supertranslations
- $\Omega^\mu{}_\nu$ generates global rotations

- Kinematical states:

$$\Psi = \sum_i \Psi_i^{bulk}[Q_i, \Omega_i] \otimes |Q_i, \Omega_i\rangle$$

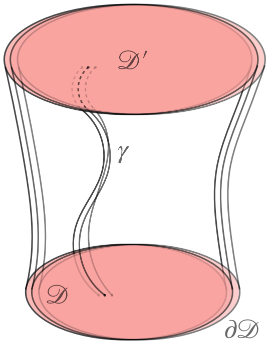
- Projector onto physical states

$$\boldsymbol{P} = \int \mathscr{D}N \exp(-\mathrm{i}N^\mu C_\mu)$$

- Defines multi-fingered boundary Schrödinger equation

$$\mathrm{i}\hbar \frac{\delta}{\delta Q^\mu} \Psi[Q, \Omega] = H_\mu \Psi[Q, \Omega]$$

- Similar for internal and global Lorentz transformations



Summary

We discussed:

1. In gravity, local subsystems are always open systems. Metriplectic geometry provides new framework to think about open systems from algebraic perspective.
2. Edge modes, Coulombic fields and radiation field in linearised gravity.
3. Multi-fingered Schrödinger equation at the boundary of spacetime.
4. Sending boundary to infinity $\rightsquigarrow$ quantum reference frames for asymptotic symmetries.