Metriplectic geometry for gravitational subsystems

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LOOPS'22, Lyon, France

21-07-2021

In gravity, every subsystem is an open system

Outline

We are witnessing a shift of perspective (at this conference) from global aspects of quantum gravity to a more local description of gravitational subsystems [Freidel, Pranzetti, Barbero, Campiglia, Geiller, Carrozza, Hoehn, Livine, Lewandowski, Odak, Margalef, Peraza, Schiavina, ww,...].

I will pick two results of the programme thus far.

1. Immirzi parameter, radiative phase space on the lightcone
2. Metriplectic geometry for gravitational subsystems

*ww, Gravitational $SL(2,\mathbb{R})$ Algebra on the Light Cone, JHEP **57** (2021), `arXiv:2104.05803`.
*Viktoria Kabel and ww, Metriplectic geometry for gravitational subsystems, (2022), `arXiv:2206.00029`.

Immirzi parameter, boundary symmetries on the lightcone

Null surface geometry

Signature $(0{+}{+})$ metric.

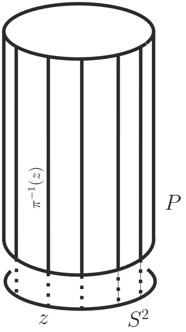
$$q_{ab} = \delta_{ij} e^i{}_a e^j{}_b, \qquad i,j = 1,2.$$

Parametrisation of the dyad

$$\boxed{e^i = \Omega\, S^i{}_j\, e^j_{(o)}.}$$

Choice of time:

$$\partial^b_U \nabla_b \partial^a_U = -\frac{1}{2}(\Omega^{-2}\frac{\mathrm{d}}{\mathrm{d}U}\Omega^2)\partial^a_U$$



Kinematical phase space for radiation: $\mathscr{P}_{kin} = \mathscr{P}_{abelian} \times T^*SL(2,\mathbb{R})$.

$$\Theta_{\mathcal{N}} = \frac{1}{8\pi G}\int_{\mathcal{N}} d^2v_o \wedge \left[p_K \,\mathbb{d}\widetilde{K} + \frac{1}{\gamma}\Omega^2\,\mathbb{d}\widetilde{\Phi} + \widetilde{\Pi}^i{}_j \left[S\mathbb{d}S^{-1}\right]^j{}_i \right] + \textit{corner term.}$$

Abelian variables:

$U(1)$ connection: $\widetilde{\Phi}$, area: $\Omega^2\, d^2v_o$, lapse: $\widetilde{K} := \text{đ}U$, expansion: p_K.

Upon imposing 2nd-class constraints: Dirac bracket for radiative modes

$$\left\{S^i{}_m(x), S^j{}_n(y)\right\}^* = -4\pi G\,\Theta(U_x,U_y)\,\delta^{(2)}(\vec{x},\vec{y})\,\Omega^{-1}(x)\,\Omega^{-1}(y) \\ \times\left[\mathrm{e}^{-2\,\mathrm{i}\,(\Delta(x)-\Delta(y))}\left[X S(x)\right]^i{}_m \left[\bar{X}S(y)\right]^j{}_n + \mathrm{cc.}\right].$$

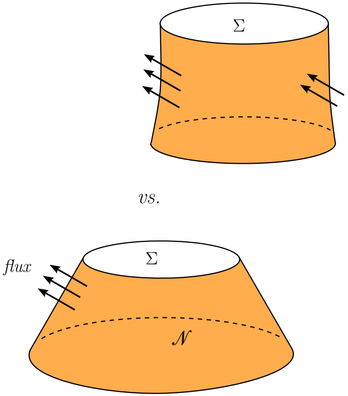
Gauge symmetries:

1. $U(1)$ gauge symmetry with $U(1)$ holonomy $h(x) = \mathrm{e}^{-\mathrm{i}\Delta(x)}$
2. vertical diffeomorphisms along null generators

Metriplectic geometry for gravitational subsystems

To understand the time evolution of a gravitational subsystem,
two choices must be made.

- Choice of time: A choice must be made for how to extend the boundary of the partial Cauchy surface Σ into a worldtube $\mathscr{N}$.
- A choice must be made how to treat the flux of gravitational radiation across the worldtube of the boundary. Flux drives the time-dependence of the system.
- Metriplectic geometry is a novel algebraic framework to tackle these issues.



N.B.: In spacetime dimensions $d < 4$, there are no gravitational waves, and we can forget about the second issue. The Hamiltonian will be automatically conserved.

In Hamiltonian systems, energy is conserved

Symplectic potential and volume-form on phase space

$$\Theta = p\,\mathrm{d}q, \quad \Omega = \mathrm{d}p \wedge \mathrm{d}q.$$

Hamilton equations

$$\Omega\Big(\delta, \frac{\mathrm{d}}{\mathrm{d}t}\Big) = \delta p\,\dot{q} - \dot{p}\,\delta q = \\ = \delta p \frac{\partial H}{\partial p} + \frac{\partial H}{\partial q}\delta q = \delta H.$$

The Hamiltonian is conserved under its own flow

$$\frac{\mathrm{d}}{\mathrm{d}t}H = \Omega\Big(\frac{\mathrm{d}}{\mathrm{d}t}, \frac{\mathrm{d}}{\mathrm{d}t}\Big) = 0.$$

If we insist that there is a Hamiltonian that drives the evolution in a finite region, the standard approach is too restrictive to account for dissipation.

Three possible viewpoints

1. There is no problem: Open systems interact with their environment. There is no Hamiltonian that would be measure the gravitational energy in a finite region.

2. Treat the system as explicitly time-dependent.
   - Time dependence induced by the choice of (outer) boundary conditions.
   - Hamiltonian field equations modified (contact geometry).

$$\frac{\mathrm{d}}{\mathrm{d}t}F_t = \{H, F_t\} + \frac{\partial}{\partial t}F_t.$$

   - By fixing the outgoing flux, radiative data no longer free (highly non-local constraints).
   - Conjecture: Resulting phase space (on which this Hamiltonian operates) is the phase space of edge modes alone. *Seems too restrictive, less useful.*

3. Metriplectic geometry
   - New algebraic approach. New bracket. But many properties of Poisson manifolds lost.
   - Noether charges generate evolution for generic vector fields.
   - Takes into account dissipation.

Metriplectic geometry
work with *Viktoria Kabel*

Metriplectic geometry

Even dimensional manifold $\mathscr{P}$, equipped with a pre-symplectic two-form $\Omega(\cdot,\cdot) \in \Omega^2(\mathscr{P})$ and a signature (p,q,r) metric tensor $G(\cdot,\cdot)$.

A vector field $\mathfrak{X}_F$ is a (right) Hamiltonian vector field of some (gauge invariant) functional $F : \mathscr{P} \to \mathbb{R}$ on $(\mathscr{P}, \Omega, G)$ iff

$$\forall \delta \in T\mathscr{P} : \delta[F] = \Omega(\delta, \mathfrak{X}_F) - G(\delta, \mathfrak{X}_F).$$

The Leibniz bracket between two such functionals is given by

$$(F, G) = \mathfrak{X}_F[G].$$

The metric on phase space encodes dissipation

$$\frac{\mathrm{d}}{\mathrm{d}t} H = (H, H) = -G(\mathfrak{X}_H, \mathfrak{X}_H).$$

*Morrison; Kaufman (1982-); Grmela, Göttinger (1997); Guha (2002); Holm, Stanley (2003);...

Metriplectic geometry and extended phase space

Following Freidel, Ciambelli, Leigh, we work on an extended pre-symplectic phase space.

A point on the extended pre-symplectic phase space is labelled by a Einstein metric g_{ab} and choice of coordinate functions x^μ.

Maurer – Cartan form for diffeomorphisms

$$\mathbb{X}^a = \left[\frac{\partial}{\partial x^\mu}\right]^a \mathbb{d}x^\mu .$$

Extended pre-symplectic current

$$\delta[L] \approx \mathrm{d}[\vartheta(\delta)],$$

$$\vartheta_{ext} = \vartheta - \vartheta(\mathscr{L}_{\mathbb{X}}) + \mathbb{X} \lrcorner L = \vartheta - \mathrm{d}q_{\mathbb{X}} .$$

Noether charge and Noether charge aspect

$$Q_{\mathbb{X}} = \oint_{\partial\Sigma} q_{\mathbb{X}} = \int_{\Sigma} \big(\vartheta(\mathscr{L}_{\mathbb{X}}) - \mathbb{X} \lrcorner L\big).$$

*L. Freidel, A canonical bracket for open gravitational system, (2021), arXiv:2111.14747.
*L. Ciambelli, R. Leigh, Pin-Chun Pai, Embeddings and Integrable Charges for Extended Corner Symmetry, Phys. Rev. Lett. **128** (2022), arXiv:2111.13181.

The coordinate functions $x^\mu : U \subset \mathscr{M} \to \mathbb{R}^4$ are now part of phase space.

Variations of coordinate functions will only contribute a corner term to the extended pre-symlpleictc two-form.

Vector fields that are determined by their component functions $\xi^\mu(x)$ become field dependent vector fields.

$$\xi^a = \xi^\mu(x)\partial^a_\mu,$$

$$\delta[\xi^a] = [\mathbb{X}(\delta), \xi]^a.$$

Extended pre-symplectic structure on the covariant phase space [Freidel; Ciambelli, Leigh]

$$\Omega_{ext}(\delta_1, \delta_2) = \Omega(\delta_1, \delta_2) + Q_{[\mathbb{X}(\delta_1), \mathbb{X}(\delta_2)]} + \oint_{\partial\Sigma} \mathbb{X}(\delta_{[1}) \lrcorner \vartheta(\delta_{2]})$$

Super metric on phase space [Viktoria Kabel, ww]

$$G(\delta_1, \delta_2) = -\oint_{\partial\Sigma} \mathbb{X}(\delta_{(1}) \lrcorner \vartheta(\delta_{2)})$$

Leibniz bracket on extended phase space,

$$\delta[F] = \Omega_{ext}(\delta, \mathfrak{X}_F) - G(\delta, \mathfrak{X}_F),$$
$$(F, G) = \mathfrak{X}_F[G].$$

Noether charge and Hamiltonian vector fields

On the extended phase space, the Lie derivative $\mathscr{L}_\xi$ is a Hamiltonian vector field with respect to the Leibniz structure.

The corresponding generator is the Noether charge,

$$\delta[Q_\xi] = \Omega_{ext}(\delta, \mathscr{L}_\xi) - G(\delta, \mathscr{L}_\xi).$$

Leibniz bracket captures dissipation

$$(Q_\xi, Q_\xi) = -\oint_{\partial\Sigma} \xi \lrcorner \vartheta(\mathscr{L}_\xi).$$

But violates Jacobi identity and skew-symmetry of Poisson bracket

$$\begin{aligned}(A,(B,C)) + (B,(C,A)) + (C,(A,B)) &\neq 0,\\ (A,B) &\neq (B,A).\end{aligned}$$

Summary

Summary

We discussed two results:

1. Immirzi parameter mixes $U(1)$ frame rotations and dilations on the null cone. **Provides a geometric explanation for LQG discreteness of geometry.**
2. New bracket: Leibniz bracket consists of skew-symmetric-symmetric (symplectic) and symmetric (metric) part. Symmetric part is a corner term that describes dissipation.