Barnich–Troessaert Bracket, Edge Modes and All That

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16-06-2021

Outline

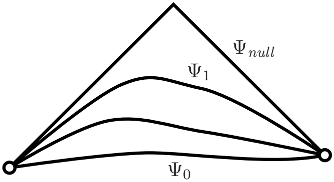
1. Introduction and Motivation
2. Bulk plus boundary field theories and covariant phase space
3. Why the Barbero – Immirzi parameter matters
4. Outlook and conclusion

Introduction and Motivation

Why quantum gravity in finite regions? Different views:

- Mere gauge fixing: Represent diffeomorphism equivalence class of states $[\Psi_0]$ by states on the light cone.

- Coarse graining: Build observables by successively gluing gravitational subsystems.



- Soft modes/edge modes: In gravity, energy, momentum, angular momentum, center of mass, supertranslations ... are analogous to charge in QED. Do we have superpositions of such charges in nature? Can we study such charge superpositions in the lab? Help us understand black hole information loss?

[Strominger, Perry; Godazgar, Harlow, Wu; Prabhu, Chandrasekaran, Flanagan, Bonga; Bodendorfer, Thiemann, Thurn; Sahlmann; Krasnov, Ashtekar, Beetle, Krishnan, Corichi, Carlip, Giddings, Freidel, Donnelly, Speranza, Riello, Geiller, Livine, Dittrich, Pranzetti, Grumiller, Seraj, Barnich, Compère,...]

To understand how gravity couples to boundaries, it is useful to work with differential forms rather than tensors since there is a natural notion of projection onto the boundary, namely the pull-back $\varphi^* : T^*M \to T^*(\partial M)$, which does not require a metric.

Fundamental configuration variables

$$g_{ab} = \eta_{\alpha\beta} e^{\alpha}{}_{a} e^{\beta}{}_{b},$$
$$\nabla \wedge \omega^{\alpha} = \mathrm{d} \wedge \omega^{\alpha} + A^{\alpha}{}_{\beta} \wedge \omega^{\beta}.$$

Palatini action

$$S[A, e] = \frac{1}{16\pi G} \int_{\mathscr{M}} *\underbrace{(e_\alpha \wedge e_\beta)}_{\Sigma_{\alpha\beta}} \wedge F^{\alpha\beta}[A] + \text{boundery terms}.$$

Symplectic potential

$$\Theta_\Sigma = \frac{1}{16\pi G} \int_{\Sigma} *\Sigma_{\alpha\beta} \wedge \mathbb{d} A^{\alpha\beta} + \text{corner terms}.$$

Charges and symmetries 1/2

Two kinds of gauge symmetries: diffeomorphisms and internal Lorentz transformations.

Lorentz transformations

$$\delta_\Lambda[e^\alpha] = \Lambda^\alpha{}_\beta e^\beta, \qquad \Lambda_{\alpha\beta} = -\Lambda_{\beta\alpha}$$
$$\delta_\Lambda[A^\alpha{}_\beta] = -\nabla\Lambda^\alpha{}_\beta.$$

Lorentz charges are integrable at full non-perturbative level.

$$\Omega_\Sigma(\delta_\Lambda,\delta)\big|_{\mathrm{EOM}} = -\delta[Q_\Lambda].$$
$$Q_\Lambda[\Sigma] = -\frac{1}{16\pi G}\oint_{\partial\Sigma} *\Sigma_{\alpha\beta}\Lambda^{\alpha\beta}.$$

NB: Such Lorentz charges do not exist in metric gravity (on the ADM phase space). Physically meaningful perhaps only if we add fermions (defects of torsion).

[Freidel, Donnelly, Speranza, Riello, Geiller, Speziale, Paoli, Oliveri, ...]

Charges and symmetries 2/2

Two kinds of gauge symmetries: diffeomorphisms and internal Lorentz transformations.

Base diffeomorphisms lifted upwards into the Lorentz bundle

$$\delta_\xi[e^\alpha] = \nabla(\xi \lrcorner e^\alpha) + \xi \lrcorner (\nabla \wedge e^\alpha),$$
$$\delta_\xi[A^\alpha{}_\beta] = \xi \lrcorner F^\alpha{}_\beta\,.$$

Diffeomorphism charges

$$\Omega_\Sigma(\delta_\xi, \delta)\big|_{\mathrm{EOM}} = \frac{1}{16\pi G}\oint_{\partial\Sigma} \xi \lrcorner * \Sigma_{\alpha\beta} \wedge \delta A^{\alpha\beta} \overset{?}{=} -\delta[P_\xi].$$

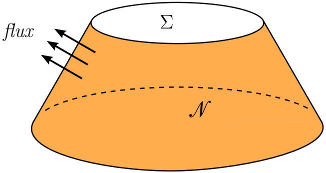
Trivially integrable at linear order in perturbations

$$e^\alpha = \mathring{e}^\alpha + f^\alpha \equiv \mathring{e}^\alpha + f^\alpha{}_\beta\, \mathring{e}^\beta, \quad f_{\alpha\beta} = f_{\beta\alpha},$$
$$P_\xi = \frac{1}{8\pi G}\oint_{\partial\Sigma} \xi \lrcorner * \mathring{\Sigma}_{\alpha\beta} \wedge \mathring{\nabla}^{[\alpha} f^{\beta]}.$$

NB: for an asymptotic time translation $\xi^a = \left[\frac{\partial}{\partial x^0}\right]^a$, the linearised charge P_ξ returns the ADM mass for a linearised solution $f_{\alpha\beta} = \mathcal{O}(r^{-1})$ around $\mathring{e}^\alpha = \mathrm{d}x^\alpha$.

- In gravity, time evolution $t \to t + \varepsilon$ can be understood as a large gauge transformation.
- It seems reasonable to expect the Hamiltonian is the generator for such a gauge transformation:

$$H[\Sigma] \equiv P_\xi[\Sigma] \stackrel{?}{=} \oint_{\partial\Sigma} d^2v^a\, \xi^b T_{ab}[?].$$



- We assume that P_ξ generates the symmetry algebra

$$\{P_\xi, P_{\xi'}\} = -P_{[\xi,\xi']} + c[\xi,\xi'].$$

- However, that's at odds with the fact that a system may loose mass via gravitational radiation

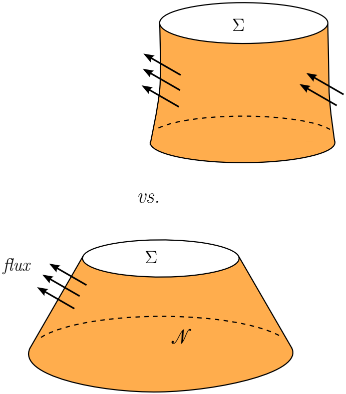
$$\left.\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}Mc^2 = \frac{\mathrm{d}}{\mathrm{d}t}H = \{H,H\} = 0,\\ = -\frac{1}{4\pi G}\oint_{S^2_t} d^2\Omega\, |\dot{\sigma}^0|^2 \leq 0.\end{aligned}\right\} \lightning$$

- ... unless, we allow for an explicit time dependence in the Hamiltonian ...

Subsystems as evolving regions in space

To characterise a gravitational subsystem,
two choices must be made.

- A choice must be made for how to extend the boundary of the partial Cauchy hypersurface Σ into a worldtube $\mathscr{N}$.
- A choice must be made for what is the flux of gravitational radiation across the worldtube of the boundary, i.e. a (background field, c-number) that drives the time-dependence of the Hamiltonian.



N.B.: In spacetime dimensions $d < 4$, there are no gravitational waves, and we can forget about the second issue. The Hamiltonian will be automatically conserved.

Bulk plus boundary field theory

Bulk plus boundary action

General action for coupled bulk plus boundary field theory

$$S=\int_{\mathscr{M}} L[\Phi,\mathrm{d}\Phi]+\int_{\mathscr{B}}\ell[\Phi,\varphi,\mathrm{d}\varphi|\sigma].$$

Fundamental configuration variables:

- bulk variables: $\Phi\in\Omega^{|\Phi|}(\mathscr{M}:\mathbb{V}_{bulk})$,
- bulk variables: $\varphi\in\Omega^{|\varphi|}(\mathscr{M}:\mathbb{V}_{bndry})$,
- boundary sources: σ,
- Covariance: for every diffeomorphism $\alpha\in\mathrm{Diff}(\mathscr{M}:\mathscr{M})$.

$$L\big[\alpha^*\Phi,\alpha^*\mathrm{d}\Phi\big](x)=L\big[\Phi,\mathrm{d}\Phi\big]\big(\alpha(x)\big),$$
$$\ell\big[\alpha^*\Phi,\alpha^*\varphi,\alpha^*\mathrm{d}\varphi\big|\alpha^*\sigma\big](x)=\ell\big[\Phi,\varphi,\mathrm{d}\varphi\big|\sigma\big]\big(\alpha(x)\big).$$

In $2+1$, the boundary source is simply the conformal metric $\sigma\equiv\sqrt{q}q^{ab}$. In higher dimensions, σ describes also the flux of gravitational radiation crossing the boundary.

Bulk plus boundary field equations

Bulk kinetic momentum: $\Pi_\Phi := \mathrm{d}\Phi$,

Bulk variations:

$$\delta[L] =: \frac{\partial L}{\partial \Phi} \wedge \delta\Phi + \frac{\partial L}{\partial \Pi_\Phi} \wedge \delta\Pi_\Phi = \\ = (\mathrm{EOM})(\delta) + \mathrm{d}\big[\Theta_{bulk}(\delta)\big].$$

Field equations and pre-symplectic current:

$$(\mathrm{EOM})(\delta) = \left[\frac{\partial L}{\partial \Phi} - (-1)^{d-|\Phi|}\mathrm{d}\Big[\frac{\partial L}{\partial \Pi_\Phi}\Big]\right] \wedge \delta\Phi,$$

$$\Theta_{bulk}(\delta) = (-1)^{d-|\Phi|}\frac{\partial L}{\partial \Pi_\Phi} \wedge \delta\Phi \equiv P_\Phi \wedge \delta\Phi.$$

Bulk plus boundary field equations

Boundary kinetic momentum: $\pi_\phi := \mathrm{d}\phi$,

Bulk variations:

$$\delta[\ell] =: \frac{\partial \ell}{\partial \Phi} \wedge \delta\Phi + \frac{\partial \ell}{\partial \varphi} \wedge \varphi + \frac{\partial \ell}{\partial \pi_\varphi} \wedge \delta\pi_\varphi + \frac{\partial \ell}{\partial \sigma} \wedge \delta\sigma.$$
$$= -\Theta_{glue}(\delta) + (\mathrm{eom})(\delta) - \mathrm{d}\big[\vartheta(\delta)\big] + \Theta_{source}(\delta).$$

Boundary field equations and pre-symplectic currents:

$$(\mathrm{eom})(\delta) = \left[\frac{\partial \ell}{\partial \varphi} + (-1)^{d-|\varphi|}\,\mathrm{d}\left[\frac{\partial \ell}{\partial \pi_\varphi}\right]\right] \wedge \delta\varphi,$$
$$\vartheta(\delta) = (-1)^{d-|\varphi|}\,\frac{\partial \ell}{\partial \pi_\varphi} \wedge \delta\varphi \equiv p_\varphi \wedge \delta\varphi.$$

Gluing conditions: linking boundary field theory to the field theory in the bulk

$$\text{at } \mathscr{B} : \Theta_{bndry}(\delta) = \Theta_{glue}(\delta).$$

Pre-symplectic structure on a partial Cauchy surface Σ anchored at the boundary $\mathscr{B}$: $\mathscr{B} \supset \partial\Sigma$,

$$\Theta_\Sigma = \int_\Sigma P_\Phi \wedge \mathbb{d}\Phi + \oint_{\partial\Sigma} p_\varphi \wedge \mathbb{d}\varphi,$$
$$\Omega_\Sigma = \mathbb{d}\Theta_\Sigma.$$

Quasi-Hamiltonian via Legendre transformation

$$H_\xi[\Sigma] = \Theta_\Sigma(\mathscr{L}_\xi) - \int_\Sigma \xi \lrcorner L + \oint_{\partial\Sigma} \xi \lrcorner \ell.$$

Hamiltonian depends on boundary sources

$$\delta\left[H_\xi[\Sigma]\right] = -\Omega_\Sigma(\mathscr{L}_\xi, \delta) + H_{\delta\xi}[\Sigma] + \oint_{\partial\Sigma} \xi \lrcorner \Theta_{source}(\delta).$$

Example: 3d gravity with conformal boundary conditions

Bulk plus boundary action:

$$S[A,e|\xi] = \frac{1}{8\pi G}\int_{\mathscr{M}} e_i \wedge F^i[\omega] - \frac{\mathrm{i}}{2}\int_{\mathscr{B}} \Big[\xi_A \mathrm{d}z \wedge D^\omega \xi^A - \mathrm{cc.}\Big]$$

- bulk variables: triad e^i and connection ω^i,
- boundary variables: ξ^A,
- boundary sources: $\sigma = \mathrm{d}z$.

Bulk plus boundary field equations

$$F^i = \mathrm{d}\omega^i + \tfrac{1}{2}\epsilon^i{}_{jk}\omega^j \wedge \omega^k = 0,$$
$$T^i = \mathrm{d}e^i + \epsilon^i{}_{jk}\omega^j \wedge e^k = 0,$$
$$\partial^a_{\bar z} D^\omega_a \xi^A = 0 \Leftrightarrow K^a{}_a = 0,$$

Gluing condition

$$\varphi^*_{\mathscr{B}} e^i = 2\pi G\, \xi^A \xi^B \sigma_{AB}{}^i \mathrm{d}z + \mathrm{cc.}$$

Recall the differential on field space

$$\delta\left[H_\xi[\Sigma]\right] = -\Omega_\Sigma(\mathscr{L}_\xi, \delta) + H_{\delta\xi}[\Sigma] + \oint_{\partial\Sigma} \xi \lrcorner \Theta_{source}(\delta) =$$
$$= -\Omega_\Sigma(\mathscr{L}_\xi, \delta) + H_{\delta\xi}[\Sigma] + \oint_{\partial\Sigma} \xi \lrcorner \Big[\frac{\partial \ell}{\partial \sigma} \wedge \delta\sigma\Big].$$

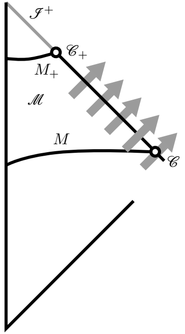
Example: In three spacetime dimensions, this can easily be made integrable. The *boundary source* is simply the conformal dyad $\sigma \equiv \mathrm{d}z$. Setting $\delta\sigma = 0$ (and $\delta\xi^a = 0$) is not a big deal in 3d.

What happens in 3+1?

To make H_ξ integrable, we choose

1. $\delta\xi^a = 0$,
2. $\delta\sigma = 0$.

What is the phase space, where these conditions are satisfied?

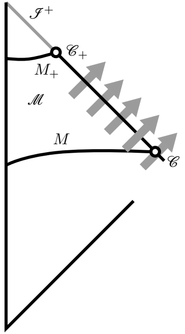


Basic assumptions: (1) $\delta\xi^a = 0$, (2) radiation has compact support and (3) phase space on M neatly splits into radiative modes and edge modes.

Crucial assumption: pre-symplectic two-form on M splits into radiative and edge modes

$$\Omega_M = \frac{1}{2}\Omega^{\alpha\beta}_{rad}[\sigma,\xi]\, \mathbb{d}\sigma_\alpha \mathbin{\wedge\!\!\!\wedge} \mathbb{d}\sigma_\beta + \frac{1}{2}\Omega^{\mu\nu}_{edge}[\sigma,\xi]\, \mathbb{d}\xi_\mu \mathbin{\wedge\!\!\!\wedge} \mathbb{d}\xi_\nu .$$

Where the radiative part is symplectomorphic (same phase space) as the portion of the radiative phase space between $\mathscr{C}$ and $\mathscr{C}_+$.



$$\Omega_{rad} = \frac{1}{2}\Omega^{\alpha\beta}_{rad}\, \mathbb{d}\sigma_\alpha \mathbin{\wedge\!\!\!\wedge} \mathbb{d}\sigma_\beta \simeq \Omega_{\mathscr{N}} .$$

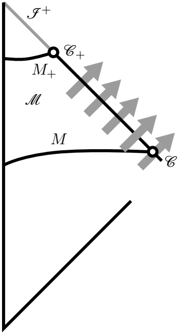
Corresponding pre-symplectic potential in terms of the asymptotic shear:

$$\Theta_{\mathscr{N}} = -\frac{1}{8\pi G}\int_{\mathscr{N}} \mathrm{d}u \wedge d^2\Omega\big(\dot{\sigma}^{(0)}\delta\bar{\sigma}^{(0)} + \mathrm{cc.}\big),$$

Constraints: fix the asymptotic shear in terms of a background field $\dot{h}$

$$\begin{aligned}\Phi_\alpha \equiv \Phi[\sigma, h](u, z, \bar{z}) = \\ = \dot{\sigma}^{(0)}(u, z, \bar{z}) - \dot{h}^{(0)}(u, z, \bar{z}) \overset{!}{=} 0,\end{aligned}$$

Where $\dot{h}$ Poisson commutes with everything.



The fundamental Poisson commutation relations imply that the constraints are second-class:

$$\{\sigma^{(0)}(x), \bar{\sigma}^{(0)}(y)\} = -4\pi G\, \Theta(x, y)\, \delta^{(2)}(x, y),$$

Dirac bracket

Dirac bracket for second class constraints $\Phi_\alpha = 0$, $\{\Phi_\alpha, \Phi_\beta\} = \Delta_{\alpha\beta}$, $\Delta^{\alpha\mu}\Delta_{\mu\beta} = \delta^\alpha_\beta$,

$$\{A, B\}^* = \{A, B\} - \{A, \Phi_\alpha\}\Delta^{\alpha\beta}\{\Phi_\beta, B\}.$$

In our case easy: the corresponding pre-symplectic two-form now simply reads

$$\Omega^*_M = \Omega_M - \Omega_{\mathcal{N}}.$$

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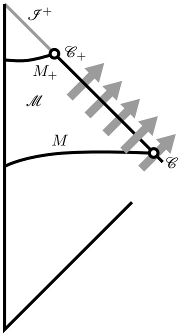
We can now simply use covariant phase space methods to compute the charge on the reduced phase space:

$$\begin{aligned}\Omega_{edge}(\delta, \mathscr{L}_\xi) &= \Omega_M(\delta, \mathscr{L}_\xi) - \Omega_{rad}(\delta, \mathscr{L}_\xi) = \\ &= \Omega_M(\delta, \mathscr{L}_\xi) - \int_{\mathscr{N}} \Big[\delta[\theta_{rad}(\mathscr{L}_\xi)] - \mathscr{L}_\xi[\theta_{rad}(\delta)]\Big] = \\ &= \Omega_M(\delta, \mathscr{L}_\xi) + \oint_{\mathscr{C}} \xi \lrcorner \theta_{rad}(\delta) - \int_{\mathscr{N}} \delta[\theta_{rad}(\mathscr{L}_\xi)].\end{aligned}$$

$$\Omega_{edge}(\delta, \mathscr{L}_\xi) = -\delta[H_\xi[M]] - \int_{\mathscr{N}} \delta[\theta_{rad}(\mathscr{L}_\xi)].$$

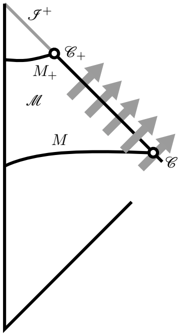
First term: Charge at $\mathscr{C}$,
Second term: Flux between $\mathscr{C}$ and $\mathscr{C}_+$.

$$\Omega_{edge}(\delta, \mathscr{L}_\xi) = -\delta[H_\xi[M_+]] \equiv -\delta H_\xi^+.$$



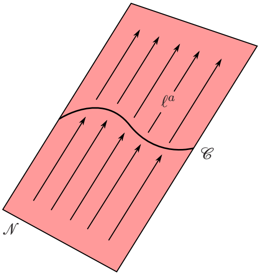
$$\{H^+_\xi, H^+_{\xi'}\}^* = \Omega_M(\mathscr{L}_\xi, \mathscr{L}_{\xi'}) - \oint_{\mathscr{C}} \Big[\xi \lrcorner \theta_{rad}(\mathscr{L}_{\xi'}) - \xi' \lrcorner \theta_{rad}(\mathscr{L}_\xi)\Big] + \int_{\mathscr{N}} \theta_{rad}\big([\xi, \bar{\xi}]\big).$$

First terms: Barnich – Trossaert Bracket,
Last term: Flux between $\mathscr{C}$ and $\mathscr{C}_+$.



Role of the Barbero – Immirzi parameter

On a null surface it is useful to work with forms rather than vectors. Given a tetrad e^α, we have a hierarchy of p-forms: $e^{\alpha_1} \wedge \cdots \wedge e^{\alpha_p}$.



- Directed area two-form $\Sigma^{\alpha\beta} = e^\alpha \wedge e^\beta$

$$\begin{pmatrix} {\Sigma^A}_B & \emptyset \\ \emptyset & -{\bar{\Sigma}_{A'}}^{B'} \end{pmatrix} = -\frac{1}{8}[\gamma_\alpha, \gamma_\beta] e^\alpha \wedge e^\beta.$$

- *On a null surface $\mathscr{N}$, there always exists a spinor $\ell^A : \mathscr{N} \to \mathbb{C}^2$ and a spinor-valued two-form ${\eta^A}_{ab} \in \Omega^2(\mathscr{N} : \mathbb{C}^2)$ such that*

$$\varphi^*_{\mathscr{N}} \Sigma_{ABab} = \ell_{(A}\eta_{B)ab}.$$

- The Lorentz invariant spin $(0,0)$ scalar $\boldsymbol{\varepsilon} = -\mathrm{i}\eta_A \ell^A$ defines the *oriented* area of any two-dimensional cross section $\mathscr{C}$ of $\mathscr{N}$

$$\mathrm{Area}[\mathscr{C}] = \int_{\mathscr{C}} \boldsymbol{\varepsilon} = -\mathrm{i} \int_{\mathscr{C}} \eta_A \ell^A.$$

Boundary term for the Holst action

Bulk plus boundary action.

- Holst action in the bulk,

$$S_{\mathscr{M}}[A,e] = \frac{\gamma+\mathrm{i}}{\gamma}\left[\frac{\mathrm{i}}{8\pi G}\int_{\mathscr{M}} \Sigma_{AB}[e]\wedge F^{AB}[A]\right] + \mathrm{cc.}$$

- $SL(2,\mathbb{C})$-invariant boundary action,

$$S_{\mathscr{N}}[A|\eta,\ell|g] = \frac{\gamma+\mathrm{i}}{\gamma}\left[\frac{\mathrm{i}}{8\pi G}\int_{\mathscr{N}} \underbrace{\eta_A\wedge\Big(D-\frac{1}{2}\varkappa\Big)\ell^A}_{\text{“pdq”}}\right] + \mathrm{cc.}$$

  The one-form $\varkappa_a\in\Omega^1(\mathscr{N})$ is the null surface analogue of the Ashtekar – Barbero connection

- bulk plus boundary action

$$S[A,e|\eta,\ell|g] = S_{\mathscr{M}}[A,e] + S_{\mathscr{N}}[A|\eta,\ell|g]$$

- boundary conditions: $\delta[g] = \delta[\varkappa_a,\ell^a,m_a]/_\sim = 0$.

Immirzi parameter twists the boundary conditions

Complex abelian connection for $U(1) \times \text{dilations}.$

$$\ell^a D_a \ell^A = \frac{1}{2}\Big(\kappa_{(\ell)} + \mathrm{i}\varphi_{(\ell)}\Big)\ell^A.$$

Boundary connection: sum of 'extrinsic curvature' and 'spin connection'.

$$\ell^a \varkappa_a = \kappa_{(\ell)} - \gamma^{-1}\varphi_{(\ell)}.$$

Boundary conditions: $\delta[\varkappa_a, \ell^a, m_a]/_\sim = 0$

- vertical diffeomorphisms $[\varphi^* \varkappa_a, \ell^a, \varphi^* m_a] \sim [\varkappa_a, \varphi_* \ell^a, m_a]$
- dilations $[\varkappa_a, \ell^a, m_a] \sim [\varkappa_a + \nabla_a f, \mathrm{e}^f \ell^a, m_a]$
- complexified conformal transformations $\lambda = \mu + \mathrm{i}\nu$:
  $[\varkappa_a, \ell^a, m_a] \sim \Big[\varkappa_a - \frac{1}{\gamma}\nabla_a \nu, \mathrm{e}^\mu \ell^a, \mathrm{e}^{\mu + \mathrm{i}\nu} m_a\Big]$
- shifts $[\varkappa_a, \ell^a, m_a] \sim [\varkappa_a + \bar{\zeta} m_a + \zeta \bar{m}_a, \ell^a, m_a]$

The equivalence class $g = [\varkappa_a, \ell^a, m_a]/_\sim$ characterises two degrees of freedom per point.

Corner term in the symplectic potential on partial Cauchy surfaces

Covariant pre-symplectic potential for the partial Cauchy surfaces:

$$\Theta_\Sigma = \left[-\frac{\mathrm{i}}{8\pi G} \oint_{\mathscr{C}} \eta_A \mathbb{d}\ell^A + \frac{\mathrm{i}}{8\pi G} \int_\Sigma \Sigma_{AB} \wedge \mathbb{d}A^{AB} \right] + \mathrm{cc.}$$

Gauge symmetries:

- Simultaneous $SL(2,\mathbb{C})$ transformations of bulk plus boundary fields.
- Small diffeomorphisms that vanish at the corner $\xi^a\big|_{\mathscr{C}} = 0$.
- $U(1)$ transformations of the boundary spinors.

The two degrees of freedom can be neatly organised into an $SL(2,\mathbb{R})$ element.

Define auxiliary $SL(2,\mathbb{R}) \ni S$ holonomy

$$\ell^a \partial_a S = \big(\varphi_{(\ell)} J + \sigma_{(\ell)} \bar{X} + \bar{\sigma}_{(\ell)} X\big) S,$$

where $(J, X, \bar{X})$ are generators of $SL(2,\mathbb{R})$

$$[J, X] = -2\mathrm{i}X,$$
$$[X, \bar{X}] = +\mathrm{i}J.$$

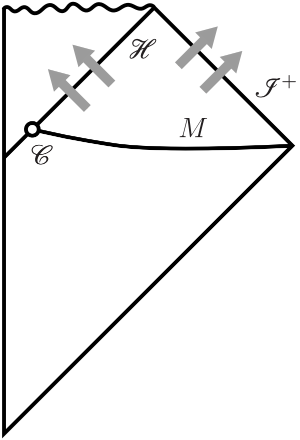
Two physical degrees of freedom encoded into homogenous space $SL(2,\mathbb{R})/U(1)$ modulo vertical diffeomorphisms.

Conclusion and Summary

Edge modes vs. radiative modes

A boundary breaks the gauge symmetries in the bulk and turns them into physical boundary modes (boundary gravitons, edge modes, pseudo Goldstone boson ...).

Physical phase space: $\mathscr{P}_M = [\mathscr{P}_M^{\textit{bulk}} \times \mathscr{P}_{\partial M}^{\textit{boundary}}]/_{\textit{gauge}}$



- In spacetime dimensions $d < 4$, there are no degrees of freedom in the bulk. Physical phase space is the phase space of boundary field theory alone.
- Treat gravity as a time dependent Hamiltonian system. Remove the radiative modes from the Cauchy hypersurface M. Encode them into auxiliary background fields. Probably enough to understand BH entropy at the full non-perturbative level.

$$\Omega_M(\delta, L_\xi) = \delta M - \Omega\delta J - \kappa\delta A = 0.$$