Quantum Bound on Gravitational Wave Luminosity

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Planck units and Planck luminosity

Simple observation: in $D=4$ spacetime dimensions, the Planck power (luminosity) is independent of $\hbar$

$$\mathscr{L}_{\mathrm{P}} = \frac{m_{\mathrm{P}} c^2}{t_{\mathrm{P}}} = \frac{\hbar^{\frac{D-4}{D-2}} c^{\frac{2D+2}{D-2}}}{G^{\frac{2}{D-2}}}.$$

Physical significance: Only in $D=4$, can we have a formula

$$\boxed{\mathscr{L}_{peak} = \mathscr{L}_{\mathrm{P}} \times f(\text{scale-independent observables}).}$$

We came close to observing such power

$$\mathscr{L}_{\mathrm{P}} = \frac{c^5}{G} \approx 3,63 \times 10^{52}\,\mathrm{W},$$

$$\mathscr{L}_{peak}\Big|_{\mathrm{GW150914}} \approx 3,6 \times 10^{49}\,\mathrm{W}.$$

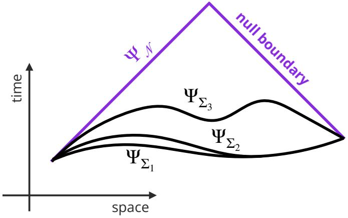
Comments:

- In $D=4$, gravitational wave luminosity non-extensive.
- N.B.: In $D=3$, Planck mass rather than luminosity $\hbar$-independent.
- Invisible in S-matrix approach, where we have a $\sqrt{\hbar G}$-expansion.

Basic setup: Gravitational subsystems on the null cone

Quantum gravity in causal regions

- Initial data: three-metric h_{ab} and extrinsic curvature $\tilde{\pi}^{ab} = d^3 v_h (K^{ab} - h^{ab} K) \sim \dot{h}_{ab}$.
- Constraints: $\mathscr{H}[h, \tilde{\pi}] = \underset{\sim}{G}_{abcd} \tilde{\pi}^{ab} \tilde{\pi}^{cd} - d^3 v_h R[h] = 0$ $\mathscr{H}_a[h, \tilde{\pi}] = D_b \tilde{\pi}^b{}_a = 0$ generate gauge redundancies on phase space.
- Gauge redundancies: states on Σ_1, Σ_2, $\ldots$ are gauge equivalent.



- Basic idea: Characterize the entire gauge equivalence class $[\Psi_{\Sigma_i}]$ by pushing the time-evolution (gauge) to its extreme.
- The boundary of the future Cauchy development of Σ_i is a null (light-like) boundary. Quantize gravity at light-like boundary. Problem simplifies. Less constraints.

[Ashtekar, Speziale, Reisenberger, Freidel, Donnelly, Ciambelli, Leigh, Geiller, Pranzetti, ..., Donney, Grumiller, Fiorucci, Ruzziconi, ..., Riello, Hoehn, Carrozza, ... , Barnich, Prabhu, Chandrasekaran, Flanagan, Compère, ...]

Null surface geometry

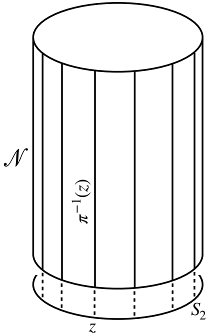
Signature $(0{+}{+})$ metric.

$$q_{ab} = \delta_{ij} e^i{}_a e^j{}_b, \qquad i, j = 1, 2.$$

Parametrisation of the dyad

$$\boxed{e^i{}_a = \Omega\, S^i{}_m \, {}^\circ e^i{}_a.}$$

- Conformal factor Ω parametrizes the overall scale.
- $S^i{}_m \in SL(2, \mathbb{R})$ determines the shape degrees of freedom.
- Fiducial background dyad $({}^\circ e^1, {}^\circ e^2) = (\mathrm{d}\vartheta, \sin\vartheta\, \mathrm{d}\varphi)$.



Choice of clock

We consider a null strip $\mathscr{N}$ with two corners as our subsystem. No unique clock along $\mathscr{N}$. Possible choice

Boundary condition at $\partial\mathscr{N} = \mathscr{C}_+ \cup \mathscr{C}_-$,

$$\mathscr{U}(\partial\mathscr{N}, z, \bar{z}) = \pm 1,$$

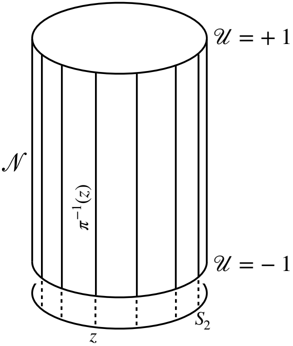
Affinity proportional to expansion

$$\partial^b_{\mathscr{U}} \nabla_b \partial^a_{\mathscr{U}} = -\frac{1}{2}(\Omega^{-2}\frac{\mathrm{d}}{\mathrm{d}\mathscr{U}}\Omega^2)\partial^a_{\mathscr{U}}$$

Parametrize physical clock $\mathscr{U}$ relative to unphysical coordinate u.

$$\partial_u \mathscr{U} \equiv \dot{\mathscr{U}} = \mathrm{e}^{\chi} > 0.$$

The *clock field* χ becomes a dynamical reference frame (QRF at quantum level).



Constraints

Gauge group and gauge fixing:

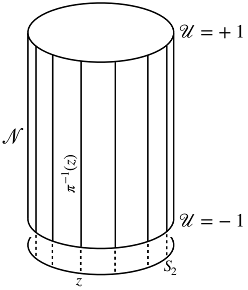
- Keep direction of light rays fixed.
- Residual diffeomorphisms: angle dependent reparametrizations of $\mathscr{U}$.
- Canonical generator on phase space: Raychaudhuri equation.

Raychaudhuri equation

$$\frac{\mathrm{d}^2}{\mathrm{d}\mathscr{U}^2}\Omega^2 = -2\,\sigma\bar{\sigma}\,\Omega^2 \mathrm{e}^{-2\chi}.$$

$SL(2,\mathbb{R})$ holonomy

$$\frac{\mathrm{d}}{\mathrm{d}u}S = \Big(\underbrace{\varphi J + \big(\sigma\bar{X} + \mathrm{cc.}\big)}_{\in\mathfrak{sl}(2,\mathbb{R})}\Big)S.$$



$SL(2,\mathbb{R})$ generators split into $U(1)$ complex structure J and shear generators:

$$[J, X] = -2\mathrm{i}\,X, \qquad [J, \bar{X}] = +2\mathrm{i}\,\bar{X}, \qquad [X, \bar{X}] = \mathrm{i}\,J.$$

Starting from the γ-action,* we obtain the symplectic structure on $\mathscr{N}$.

$$\Theta_{\mathscr{N}}(\delta) = \frac{1}{8\pi G}\int_{\mathscr{N}} d^3v_o\,\frac{\mathrm{d}}{\mathrm{d}u}\left[\Omega^2\right]\delta\chi + \\ + \frac{1}{8\pi\gamma G}\int_{\mathscr{N}} d^3v_o\,\Omega^2\delta\varphi + \int_{\mathscr{N}} d^3v_o\,\mathrm{Tr}\left(\Pi(\delta S)S^{-1}\right).$$

Key Observations:

- Radiative modes encoded into $T^*SL(2,\mathbb{R})$ symplectic structure.
- Second-class constraints for $\mathfrak{sl}(2,\mathbb{R})$ momentum $\Pi = LJ + c\bar{X} + \bar{c}X$

$$L = -\frac{1}{16\pi\gamma G}\frac{\mathrm{d}}{\mathrm{d}u}\Omega^2, \quad c = -\frac{1}{8\pi\gamma G}(\gamma+\mathrm{i})\Omega^2\sigma.$$

- Scalar constraint $H[N] = \Theta_{\mathscr{N}}(L_N) = 0$ generates a Virasoro algebra.
  See also recent results by Freidel and Ciambelli.
- Barbero–Immirzi parameter γ alters symplectic structure.
  *underlying action: $S[A,e] = \frac{1}{16\pi G}\int_{\mathscr{M}} d^4v\left[F^{\alpha\beta}{}_{\alpha\beta} - \frac{1}{2\gamma}F_{\alpha\beta\gamma\delta}\epsilon^{\alpha\beta\gamma\delta}\right].$

$SU(1,1)$ Casimir and central charge

Canonical momentum dual to the $SL(2,\mathbb{R}) \simeq SU(1,1)$ (shape) modes:

$$\Pi = LJ + c\bar{X} + \bar{c}X \in \mathfrak{su}(1,1)$$

$SU(1,1)$ Casimir in terms of the geometric data:

$$L^2 - c\bar{c} = \frac{1}{(16\pi\gamma G)^2}\Omega^4\left(\vartheta^2 - 4(1+\gamma^2)\sigma\bar{\sigma}\right).$$

What we find is:

- Bose statistics for CFT modes along light rays:
  - CFT has negative central charge.
  - Both $L^2 \leq c\bar{c}$ and $L^2 \geq c\bar{c}$ possible.
  - But resulting CFT is non-unitary.
- Fermi statistics for CFT modes along light rays:
  - CFT has positive central charge.
  - Only $L^2 \geq c\bar{c}$ *infra-Planckian* modes occur.
  - violation of unitarity can be avoided.

Luminosity bound

To ensure positive definite inner product: Fermi statistics.

This implies

$$\vartheta^2 - 4(1+\gamma^2)\sigma\bar{\sigma} \geq 0.$$

For semi-classical states (as expectation values)

$$\frac{\sigma\bar{\sigma}}{\vartheta^2} \leq \frac{1}{4}\frac{1}{1+\gamma^2}.$$

This must hold for all null hypersurfaces.

We obtain luminosity bound (power per solid angle)

$$\boxed{\mathscr{L}_{\mathrm{B}}(u,\zeta,\bar{\zeta}) = \frac{4c^5}{G}\lim_{r\to\infty}\frac{\bar{\sigma}_{(\ell)}(u,r,\zeta,\bar{\zeta})\sigma_{(\ell)}(u,r,\zeta,\bar{\zeta})}{(\vartheta_{(\ell)}(u,r,\zeta,\bar{\zeta}))^2} \leq \frac{c^5}{G}\frac{1}{1+\gamma^2}.}$$

Relationship to asymptotic observables

Utilize Bondi expansion

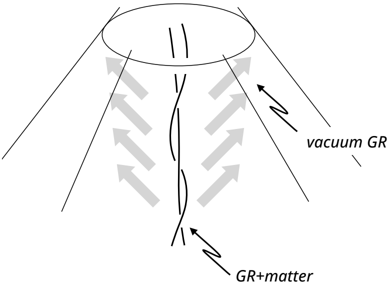
- Bondi mass loss formula

$$\dot{M}_{\mathrm{B}}(u) = -\frac{1}{4\pi G}\oint_{S^2_u \subset \mathscr{I}_+} d^2 v_o \, \dot{\sigma}^{(0)} \dot{\bar{\sigma}}^{(0)}.$$

- Falloff conditions

$$\sigma_{(\ell)}(u, r, \zeta, \bar{\zeta}) = -\frac{\dot{\sigma}^{(0)}(u, \zeta, \bar{\zeta})}{r} + \mathcal{O}(r^{-2}),$$

$$\vartheta_{(\ell)}(u, r, \zeta, \bar{\zeta}) = -\frac{2}{r} + \mathcal{O}(r^{-2}).$$



Asymptotic expansion

$$\boxed{\mathscr{L}_{\mathrm{B}}(u, \zeta, \bar{\zeta}) = \frac{4c^5}{G} \lim_{r\to\infty} \frac{\bar{\sigma}_{(\ell)}(u, r, \zeta, \bar{\zeta})\sigma_{(\ell)}(u, r, \zeta, \bar{\zeta})}{(\vartheta_{(\ell)}(u, r, \zeta, \bar{\zeta}))^2}.}$$

In the S-matrix approach, the $\mathcal{O}(r^{-1})$ term of $\vartheta_{(\ell)}$ is a commuting c-number. In the quasi-local approach, $\vartheta_{(\ell)}$ is an operator akin to LQG area operator.

Conclusion

Take home messages

- Non-perturbative quantisation of null initial data at finite distance.
  - Consistency check: Spectra for geometric observables reproduce LQG discreteness of area using CFT methods.
  - Barbero–Immirzi parameter activates otherwise irrelevant $SU(1,1)$ irreps.
- Indications for Planck luminosity bound in quantum gravity.
- Proof of principle that $r \to \infty$ and $\hbar \to 0$ may not commute.